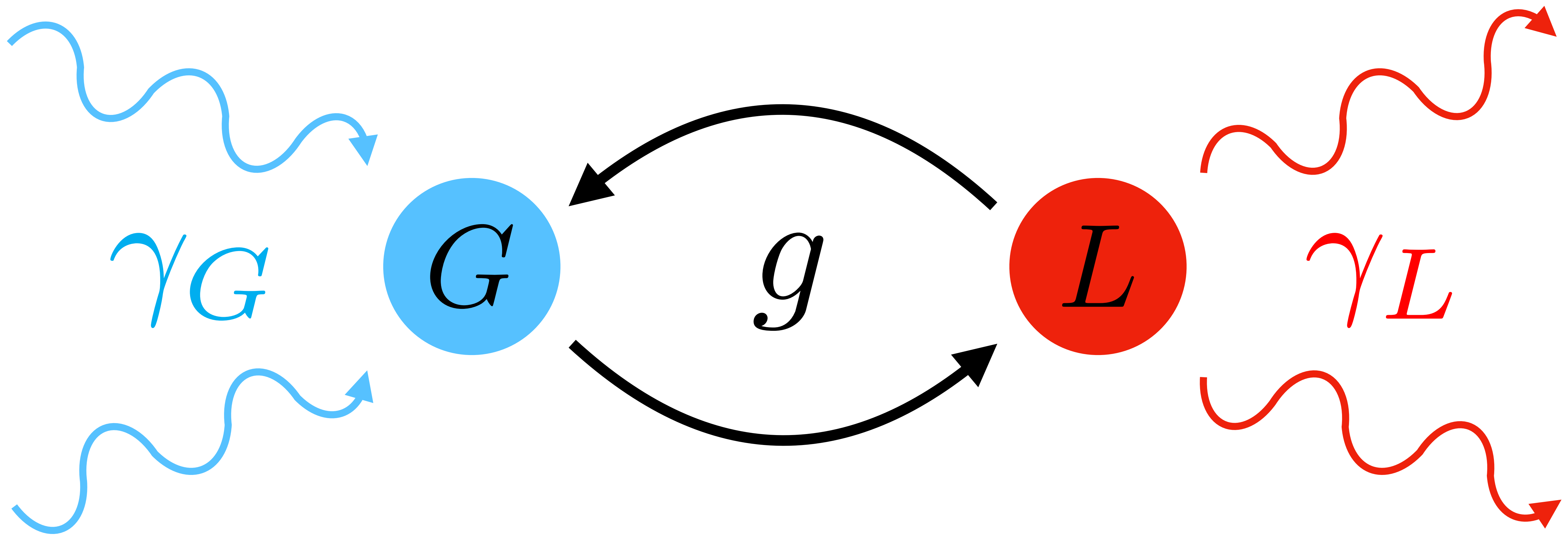
Exploring quantumness in a gain-loss system

Federico Roccati, Salvatore Lorenzo, Massimo Palma, Francesco Ciccarello

arXiv:1907.00975

Trieste Junior Quantum Days
ICTP, July 25th 2019



Teaser

- Increasing interest in **non-Hermitian** Quantum Mechanics

C. M. Bender and S. Boettcher, Phys. Rev. Lett. 80, 5243 (1998).
C. M. Bender, Rep. Prog. Phys. 70, 947 (2007).
C. M. Bender, D. C. Brody, and H. F. Jones, American Journal of Physics 71, 1095 (2003).

- **Experimental** realisations of non-Hermitian **P**arity-**T**ime-symmetric systems

R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, Nat. Phys. 14, 11 (2018).
L. Feng, R. El-Ganainy, and L. Ge, Nat. Photonics 11, 752 (2017).
S. Longhi, Euro Phys. Lett. 120, 64001 (2017).
C. E. Rueter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, Nat. Phys. 6, 192 (2010).
A. Regensburger, C. Bersch, M.-A. Miri, G. Onishchukov, D. N. Christodoulides, and U. Peschel, Na- ture 488, 167 (2012).
B. Peng, S. K. Ozdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. Fan, F. Nori, C. M. Bender, and L. Yang, Nat. Phys. 10, 394 (2014).

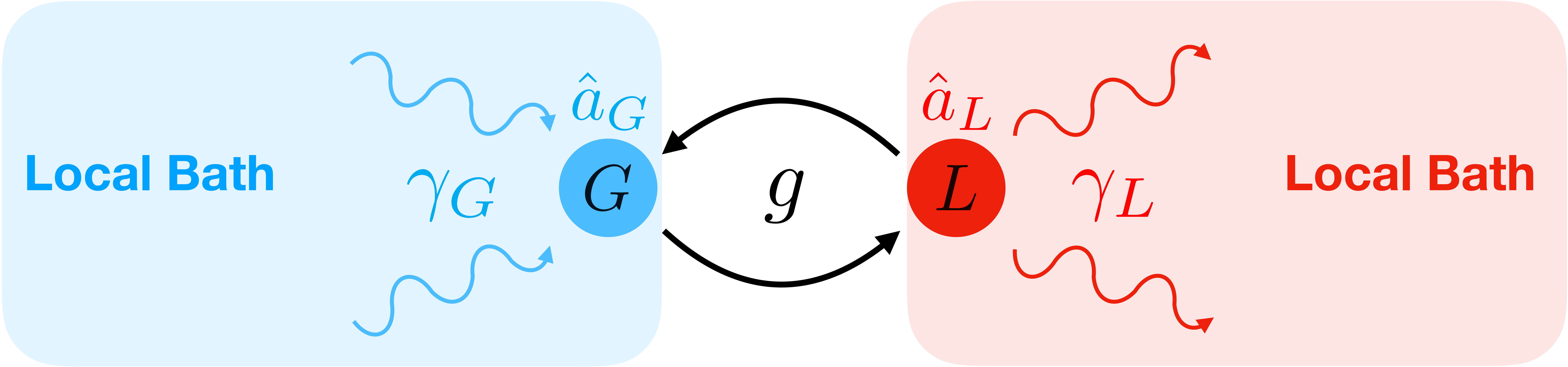
… all **classical**

- **Quantum** character of PT symmetric systems is still an **open problem**

W. Cao, X. Lu, X. Meng, J. Sun, H. Shen, and Y. Xiao, arXiv:1903.12213 [quant-ph] (2019), arXiv: 1903.12213.
Fring, Andreas, and Thomas Frith. "Eternal life of entropy in non-Hermitian quantum systems." *arXiv preprint arXiv:1905.07348* (2019).
Chakraborty, Subhadeep, and Amarendra K. Sarma. "Delayed sudden death of entanglement at exceptional points." *arXiv preprint arXiv:1906.00222* (2019).
H. Schomerus, Phys. Rev. Lett. 104, 233601 (2010). G. Yoo, H.-S. Sim, and H. Schomerus, Phys. Rev. A 84, 063833 (2011).
G. S. Agarwal and K. Qu, Phys. Rev. A 85, 031802 (2012). S. Longhi, Opt. Lett. 43, 5371 (2018).
S. Vashahri-Ghamsari, B. He, and M. Xiao, Phys. Rev. A 96, 033806 (2017) and Phys. Rev. A 99, 023819 (2019).

Gain-loss system

$$H = g\left(\hat{a}_L^\dagger \hat{a}_G + \hat{a}_L \hat{a}_G^\dagger\right).$$



S. Scheel and A. Szameit, Euro Phys. Lett. 122, 34001 (2018).
D. Dast, D. Haag, H. Cartarius, and G. Wunner, Phys. Rev. A 90, 052120 (2014).

$$\dot{\rho} = -i[H, \rho] + 2\gamma_L \mathcal{D}[\hat{a}_L]\rho + 2\gamma_G \mathcal{D}[\hat{a}_G^\dagger]\rho$$

Mean-field dynamics

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\langle\hat{a}_L\rangle\\ \langle\hat{a}_G\rangle\end{pmatrix} = \begin{pmatrix}-i\gamma_L & g\\ g & i\gamma_G\end{pmatrix}\begin{pmatrix}\langle\hat{a}_L\rangle\\ \langle\hat{a}_G\rangle\end{pmatrix}$$

- Typical example of **non-Hermitian** "Hamiltonian"
- Generally two **complex** eigenvalues, **non**-orthogonal eigenstates
- Parity Time (PT) symmetry:

R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, Nat. Phys. 14, 11 (2018).
C. E. Rueter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, Nat. Phys. 6, 192 (2010).

PARITY / TIME REVERSAL

$$\boxed{G \leftrightarrow L} + \boxed{t \to -t} \Rightarrow \boxed{\gamma_G = \gamma_L \equiv \gamma}$$

Mean-field dynamics - PT symmetry

EXACT PHASE

$$\gamma < g$$

- **Real** eigenvalues
- **<u>Non</u>**-orthogonal eigenvectors

EXCEPTIONAL POINT

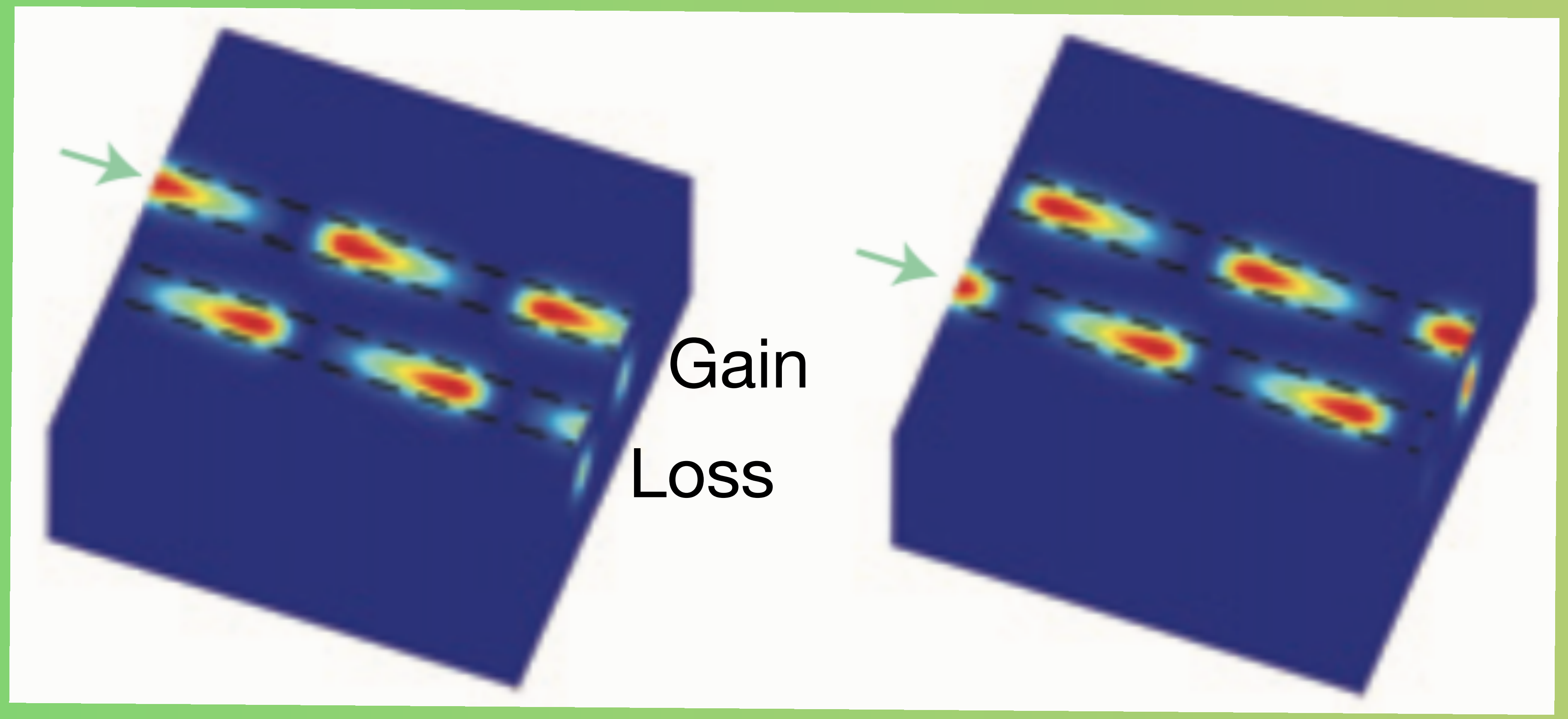
$$\gamma = g$$

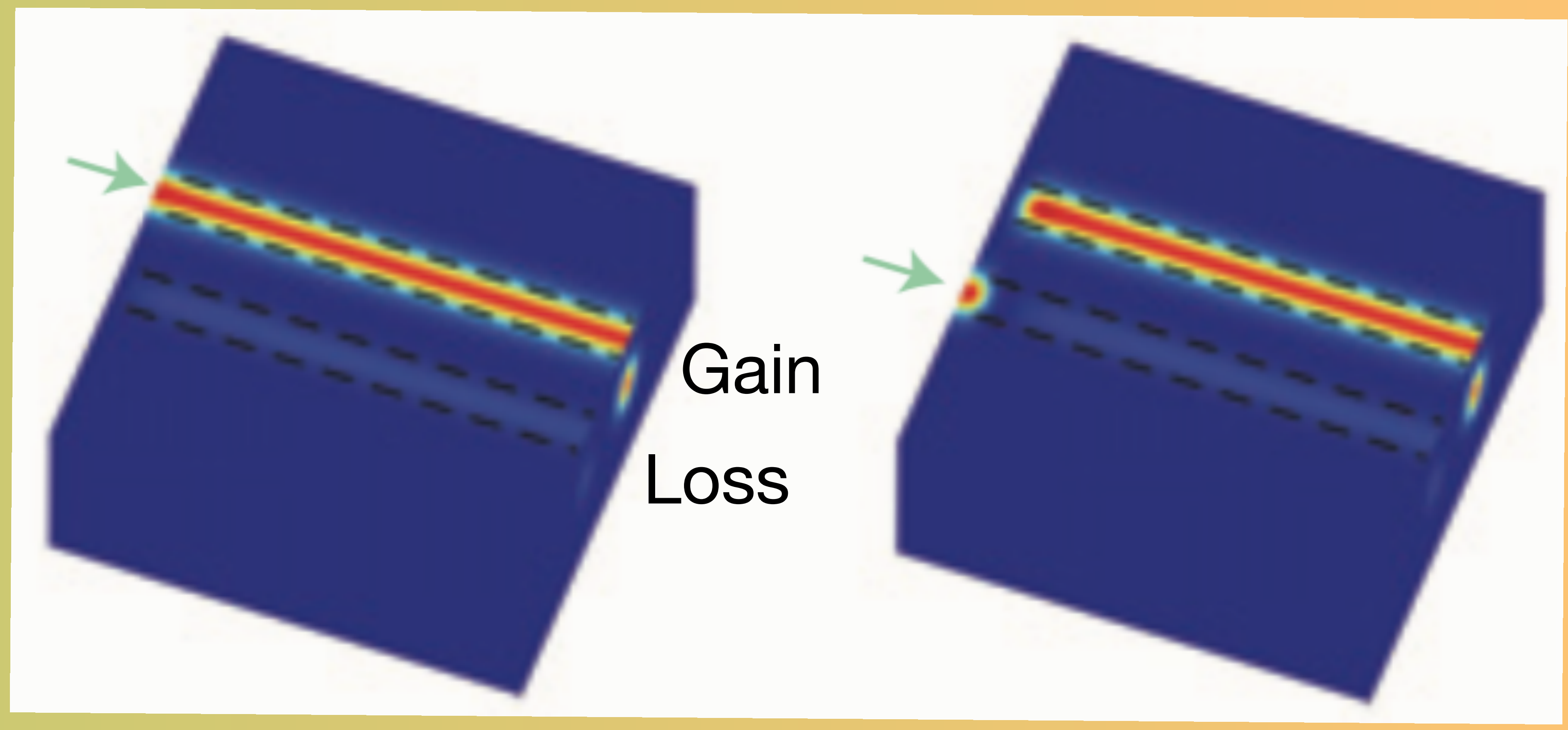
- **Coalescing** eigenvalues
- **Parallel** eigenvectors

BROKEN PHASE

$$\gamma > g$$

- **Imaginary** eigenvalues
- **<u>Non</u>**-orthogonal eigenvectors





C. E. Rueter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, Nat. Phys. 6, 192 (2010).

Second-moment dynamics

- $\hat{E} \propto (\hat{a} + \hat{a}^\dagger)$ $\longrightarrow$ **quantum uncertainties** $\propto \langle \hat{a}_L^\dagger \hat{a}_L \rangle, \langle \hat{a}_G^\dagger \hat{a}_G \rangle, \langle \hat{a}_L^\dagger \hat{a}_G \rangle, \ldots$

- Encoded in the 4 x 4 **covariance matrix:**

$$\boxed{\sigma_{ij} = \langle \hat{X}_i \hat{X}_j + \hat{X}_j \hat{X}_i \rangle}$$

$$\hat{X} = (\hat{x}_L, \hat{p}_L, \hat{x}_G, \hat{p}_G)$$

(quadratures)

C. Gardiner and P. Zoller, Quantum Noise: A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics, 3rd ed., Springer Series in Synergetics (Springer-Verlag, Berlin Heidelberg, 2004).

Second-moment dynamics

uncertainties on local fields — $\langle \hat{a}_L^\dagger \hat{a}_L \rangle, \langle \hat{a}_G^\dagger \hat{a}_G \rangle, \ldots$

cross correlations — $\langle \hat{a}_L^\dagger \hat{a}_G \rangle, \ldots$

$$\sigma = \begin{pmatrix} L & C \\ C^T & G \end{pmatrix}$$

- Time evolution:

$$\dot{\sigma} = Y\sigma + \sigma Y^T + 4D$$

C. Gardiner and P. Zoller, Quantum Noise: A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics, 3rd ed., Springer Series in Synergetics (Springer-Verlag, Berlin Heidelberg, 2004).

- Focus on **Gaussian** states: $\rho \overset{1-1}{\longleftrightarrow} \sigma$ (up to local displacement)

G. Adesso, S. Ragy, and A. R. Lee, Open Syst. Inf. Dyn. 21, 1440001 (2014).

Classical and quantum correlations

Pure states

Quantum Correlations
=
Entanglement

↓

Classical
=
Separable

Mixed states

Quantum Correlations
$\supset$
Entanglement

↓

Quantum Correlations in "classical-looking" states:
Quantum Discord

N. Korolkova and G. Leuchs, Rep. Prog. Phys. 82, 056001 (2019).

Classical and quantum correlations

Classical

Mutual Information	$S(G) + S(L) - S(GL)$
$=$	$=$
Classical Correlations	$S(G) - S(G\|L) = S(L) - S(L\|G)$

T. M. Cover and J. A. Thomas, Elements of Informa- tion Theory, 2nd ed. (Wiley-Interscience, Hoboken, N.J, 2006).

Classical and quantum correlations

Quantum

Mutual Information	$$S(\rho_G) + S(\rho_L) - S(\rho_{GL})$$ M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information: 10th Anniversary Edition, anniversary edizione ed. (Cambridge University Press, Cambridge ; New York, 2010).
Classical Correlations	**more difficult to upgrade…**

Classical and quantum correlations

- **Quantum version of Classical Correlations**

$$\mathcal{C}_{GL} = S(\rho_L) - \min_{\hat{G}_k} \sum_k p_k S(\rho_{L|k})$$

$\hat{G}_k$: local POVM measurements on G

p_k: probability of outcome k

$\rho_{L|k}$: state of L after measurement on G

Vedral V. (2017) Foundations of Quantum Discord. In: Fanchini F., Soares Pinto D., Adesso G. (eds) Lectures on General Quantum Correlations and their Applications. Quantum Science and Technology. Springer, Cham

Classical and quantum correlations

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$\hat{G}_k$: local POVM measurements on G

p_k: probability of outcome k

$\rho_{L|k}$: state of L after measurement on G

- **Quantum Discord**

$$\mathcal{D}_{GL} = \mathcal{I} - \mathcal{C}_{GL}$$

$\mathcal{I}$: Total Correlation

$\mathcal{C}_{GL}$: Classical Correlations

→ **Pure Quantum Correlations!**

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001).
L. Henderson and V. Vedral, J. Phys. A: Math. Gen. 34, 6899 (2001).
K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, Rev. Mod. Phys. 84, 1655 (2012).

Classical and quantum correlations

Quantum Discord

- Asymmetric
- Discord $\supset$ Entanglement
- **Gaussian** Discord
- **Gaussian** States

$$\mathcal{C}_{GL} \neq \mathcal{C}_{LG}$$

$$\Downarrow$$

$$\mathcal{D}_{GL} \neq \mathcal{D}_{LG}$$

K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, Rev. Mod. Phys. 84, 1655 (2012).

Classical and quantum correlations

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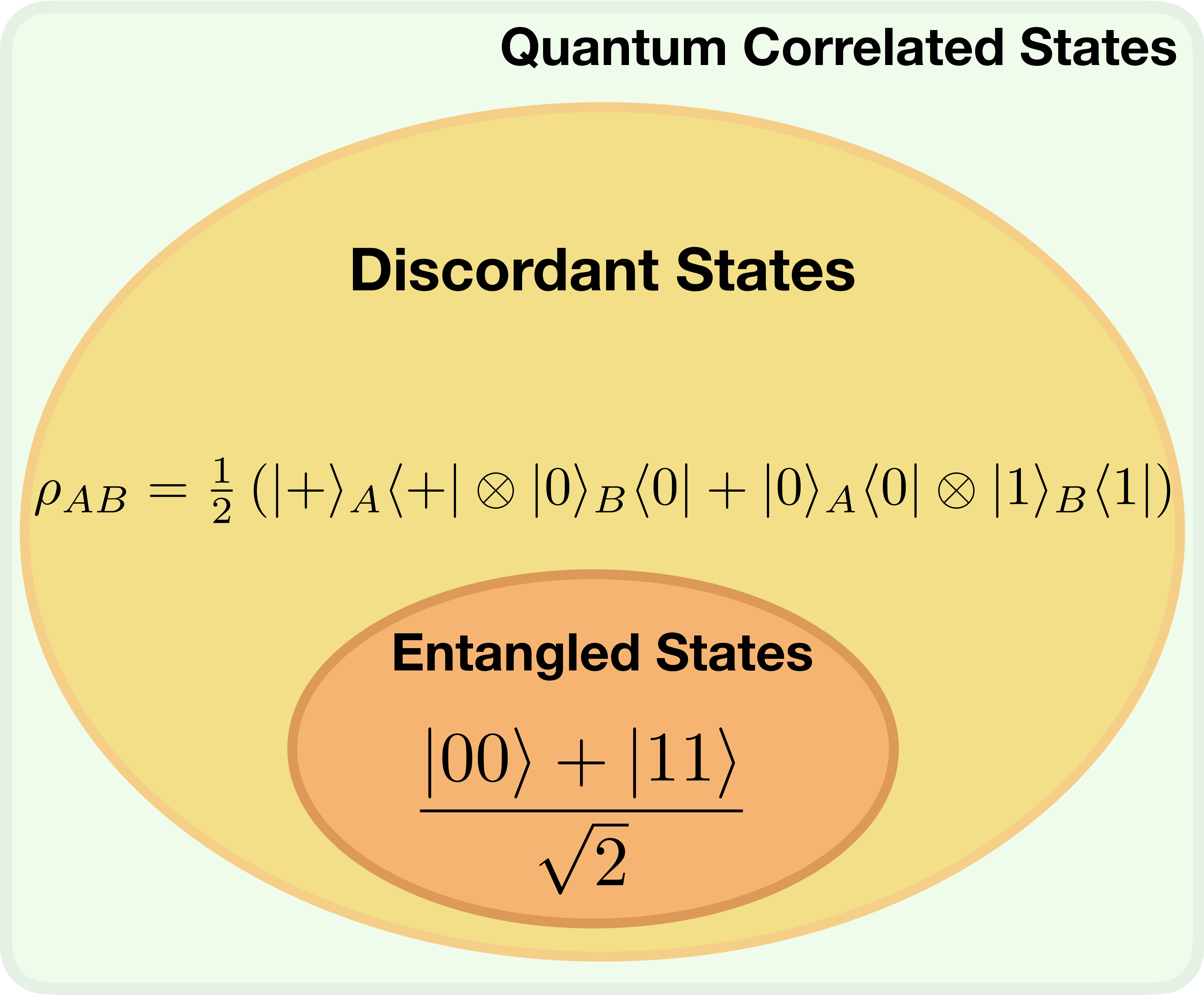


K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, Rev. Mod. Phys. 84, 1655 (2012).

Classical and quantum correlations

Quantum Discord

- Asymmetric
- Discord $\supset$ Entanglement
- **Gaussian** Discord
- **Gaussian** States

$$\mathcal{C}_{GL} = S(\rho_L) - \min_{\hat{G}_k} \sum_k p_k S(\rho_{L|k})$$

Gaussian measurements

- **Analytical Formula**

$$\mathcal{I} \rightarrow \mathcal{I}(\sigma)$$
$$\mathcal{C}_{GL} \rightarrow \mathcal{C}_{GL}(\sigma)$$
$$\mathcal{D}_{GL} \rightarrow \mathcal{D}_{GL}(\sigma)$$

P. Giorda and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010).
G. Adesso and A. Datta, Phys. Rev. Lett. 105, 030501 (2010).

- **Optimal**

S. Pirandola, G. Spedalieri, S. L. Braunstein, N. J. Cerf, and S. Lloyd, Phys. Rev. Lett. 113, 140405 (2014).

Classical and quantum correlations

Quantum Discord

- Asymmetric
- Discord $\supset$ Entanglement
- **Gaussian** Discord
- **Gaussian** States

- Separable $\Rightarrow 0 < \mathcal{D} < 1$
- Entangled $\Rightarrow \mathcal{D} > 1$

G. Adesso and A. Datta, Phys. Rev. Lett. 105, 030501 (2010).

Dynamics of quantum correlations

$$\rho_0 = |\alpha_G\rangle\langle\alpha_G| \otimes |\alpha_L\rangle\langle\alpha_L| \longleftrightarrow \sigma_0 = \mathbb{1}_4$$

No initial correlations!

G. Adesso, S. Ragy, and A. R. Lee, Open Syst. Inf. Dyn. 21, 1440001 (2014).

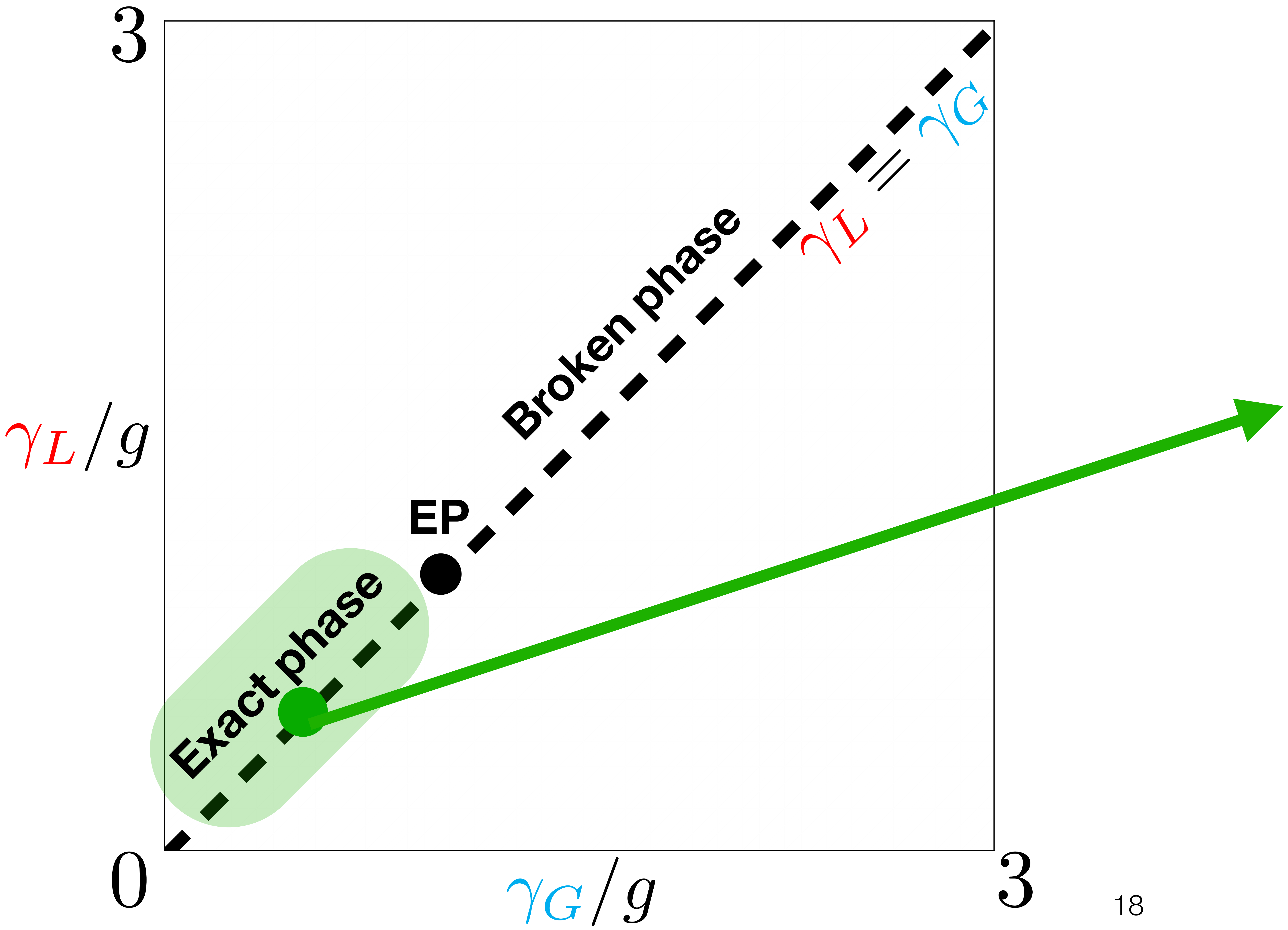
Gain / **Loss** channel:
LOCAL

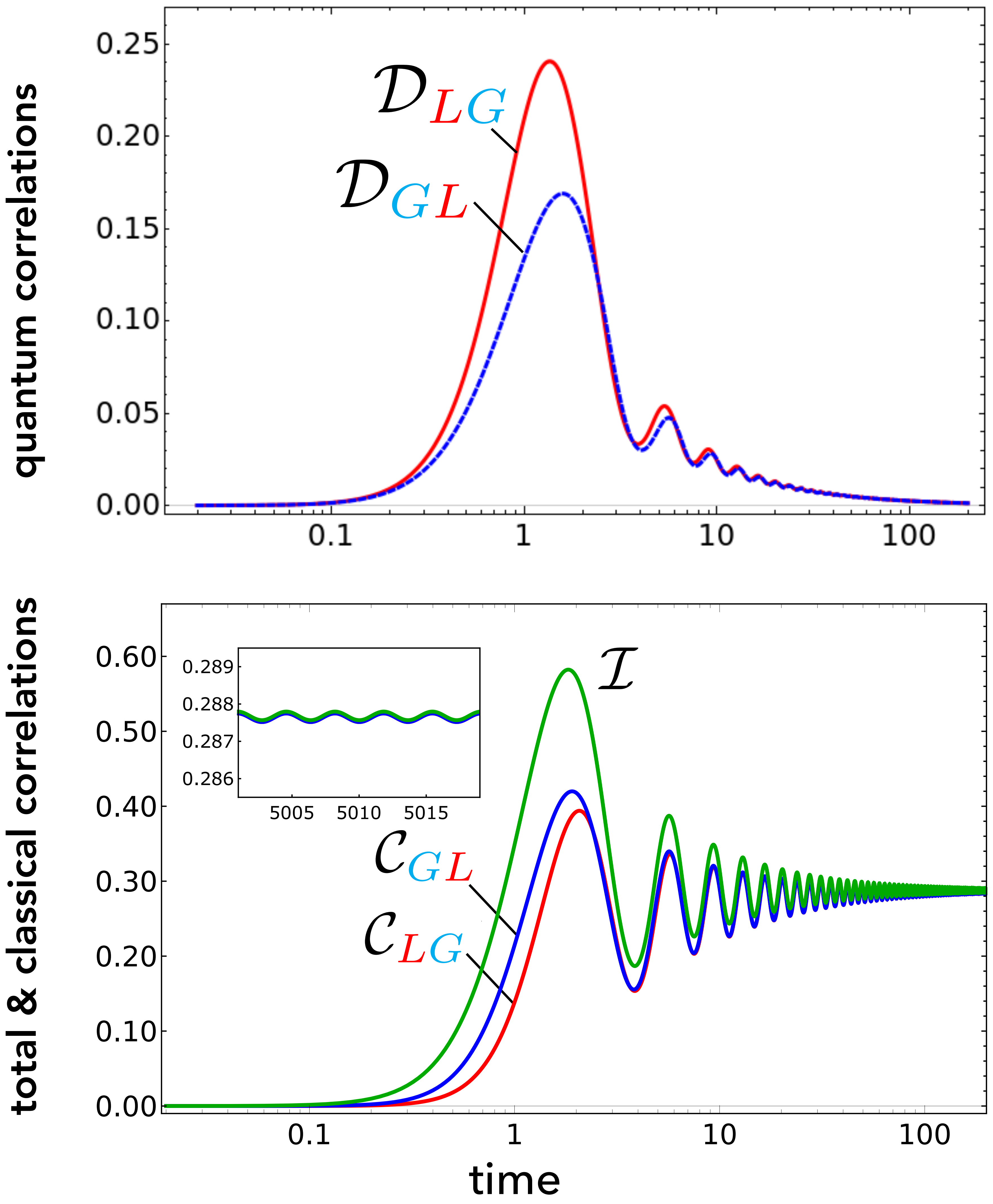
Coupling:
BEAM SPLITTER LIKE

NO ENTANGLEMENT

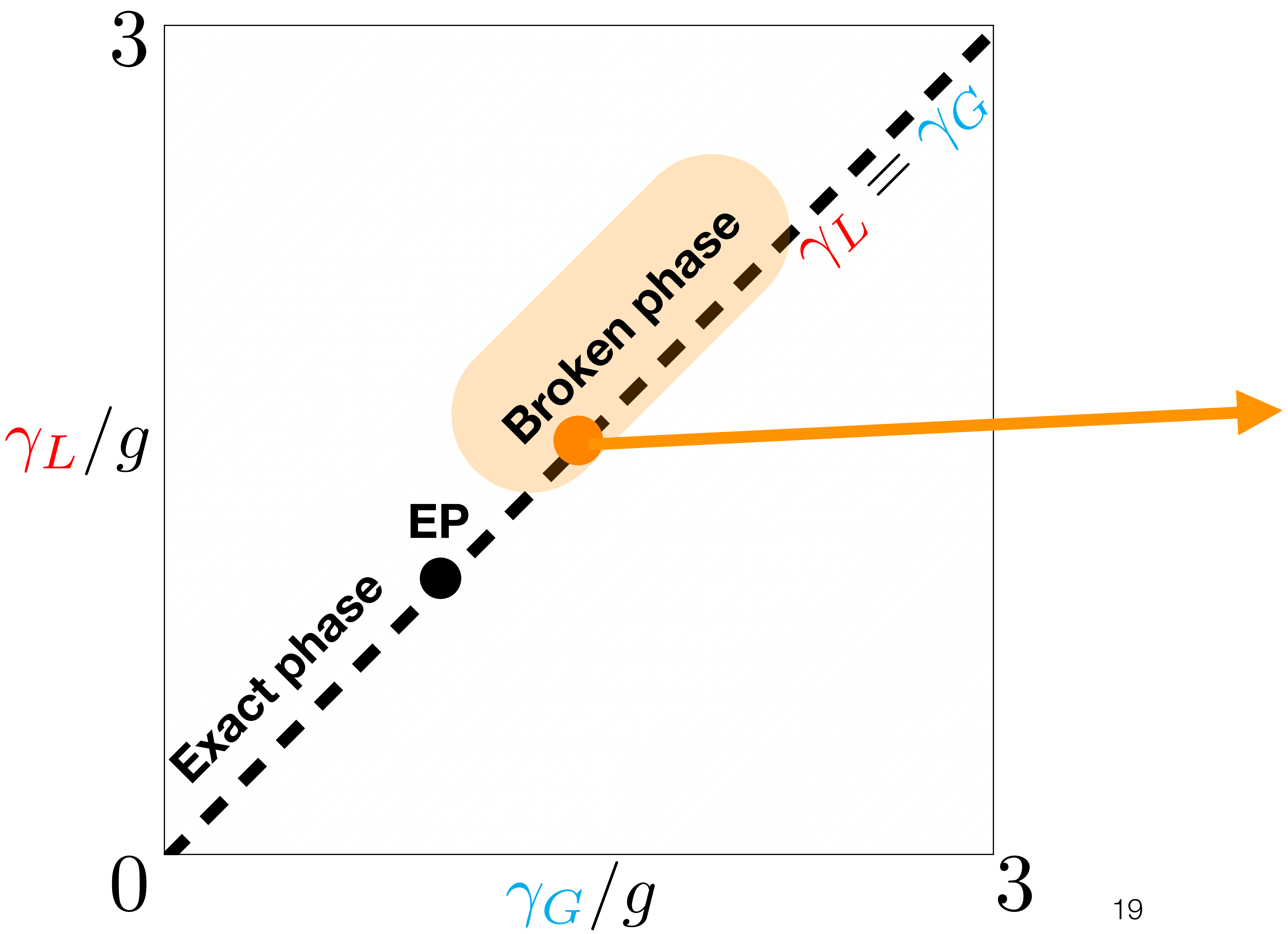
M. S. Kim, W. Son, V. Buzek, and P. L. Knight, Phys. Rev. A 65, 032323 (2002).

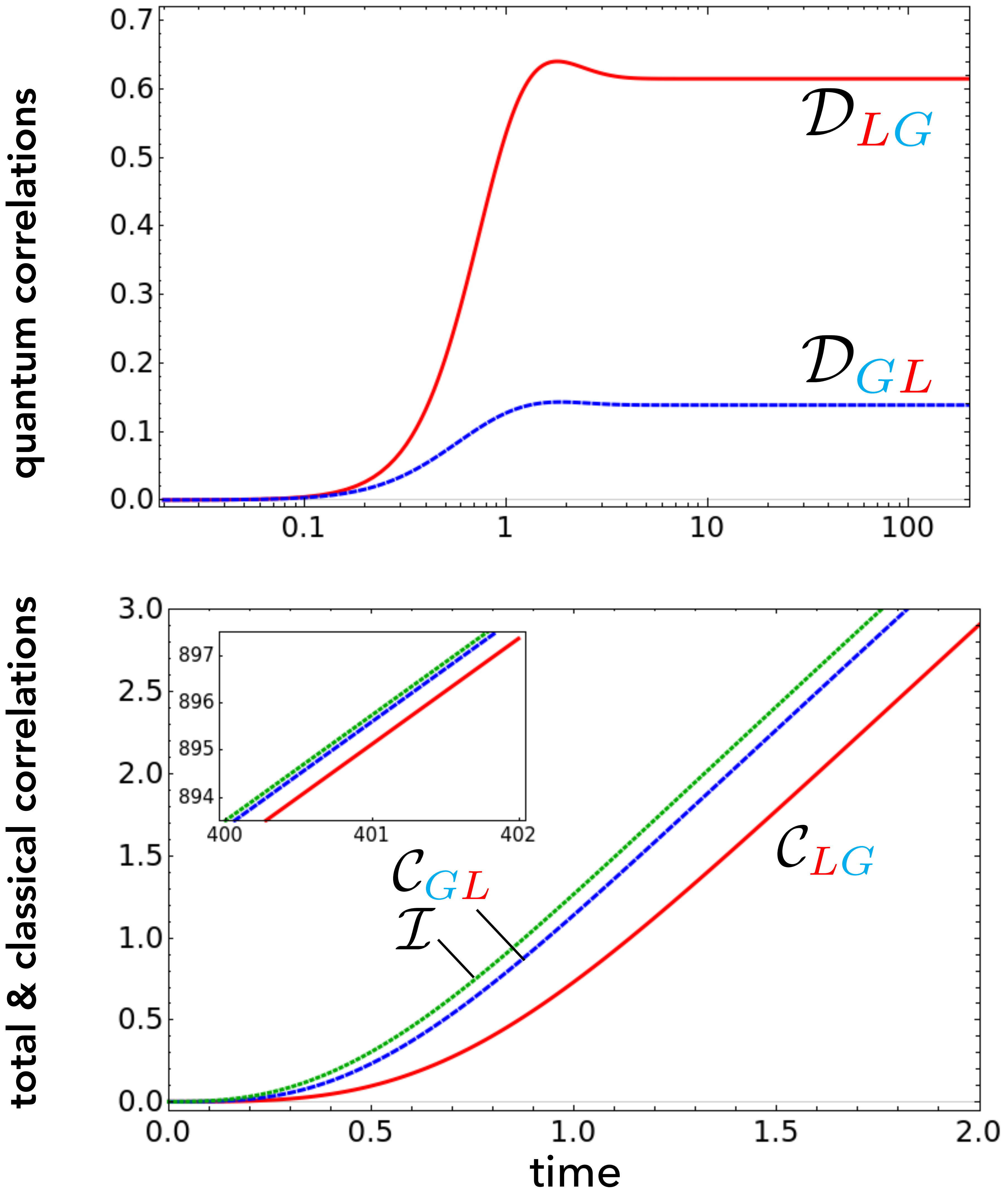
Dynamics of quantum correlations



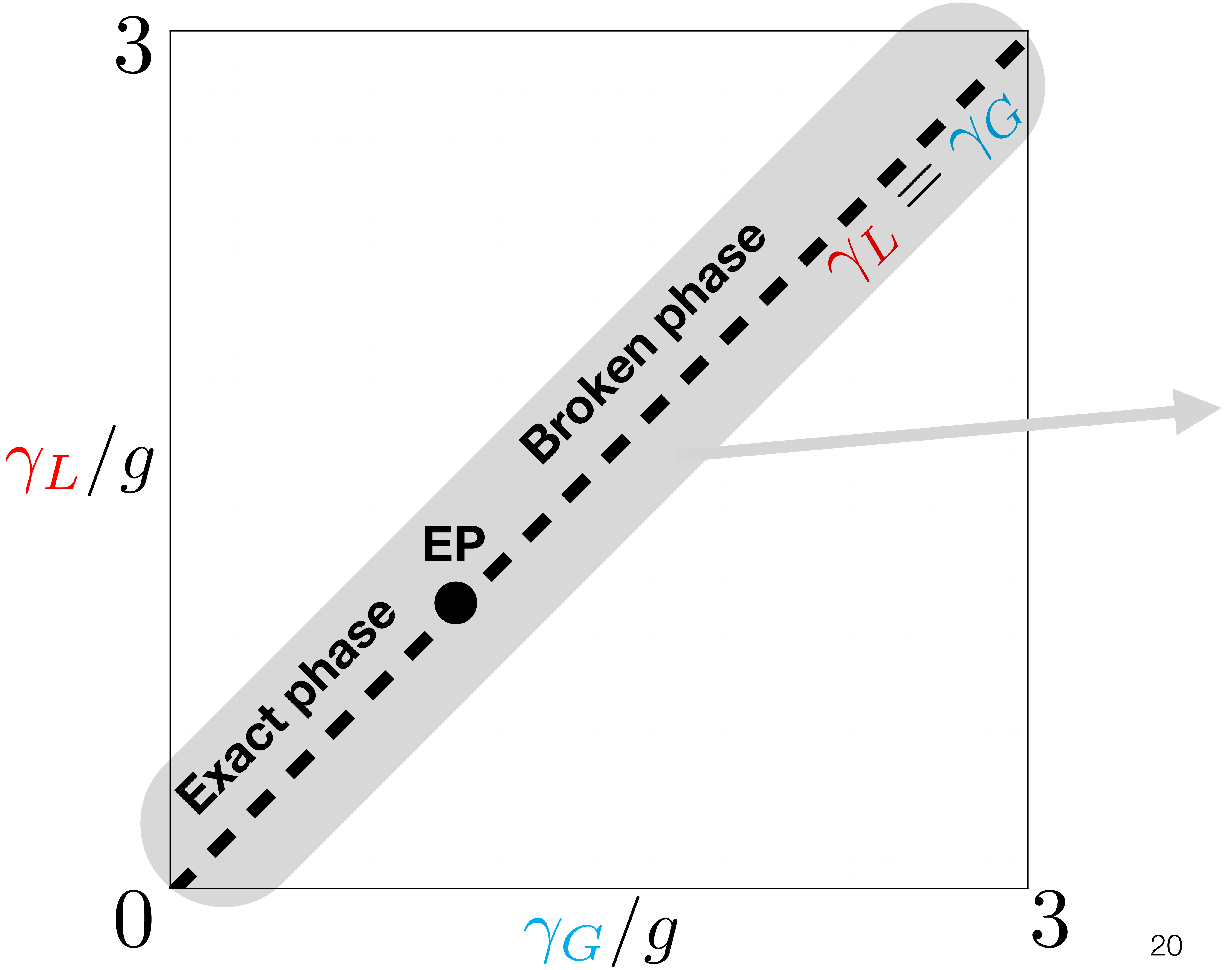


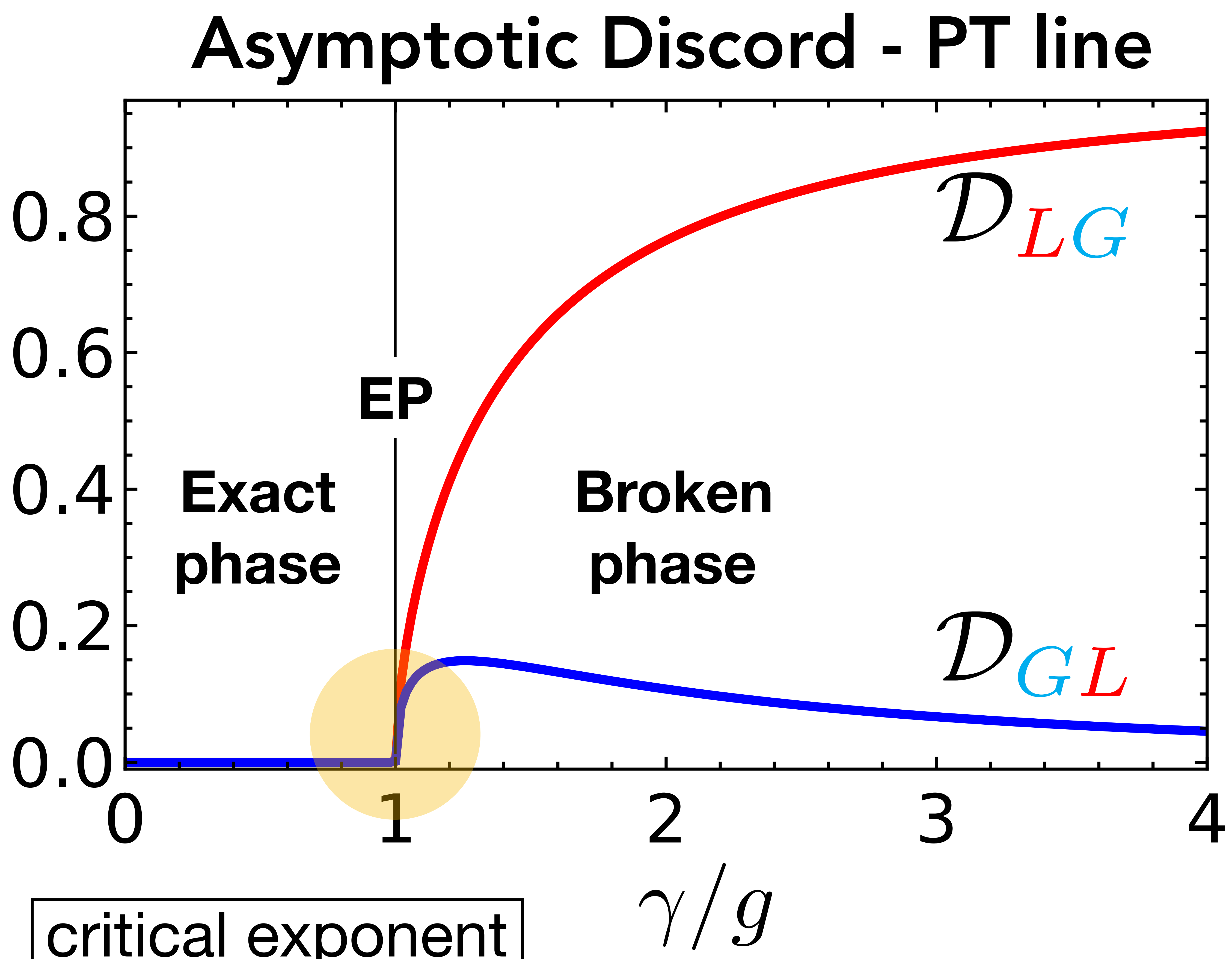
Dynamics of quantum correlations





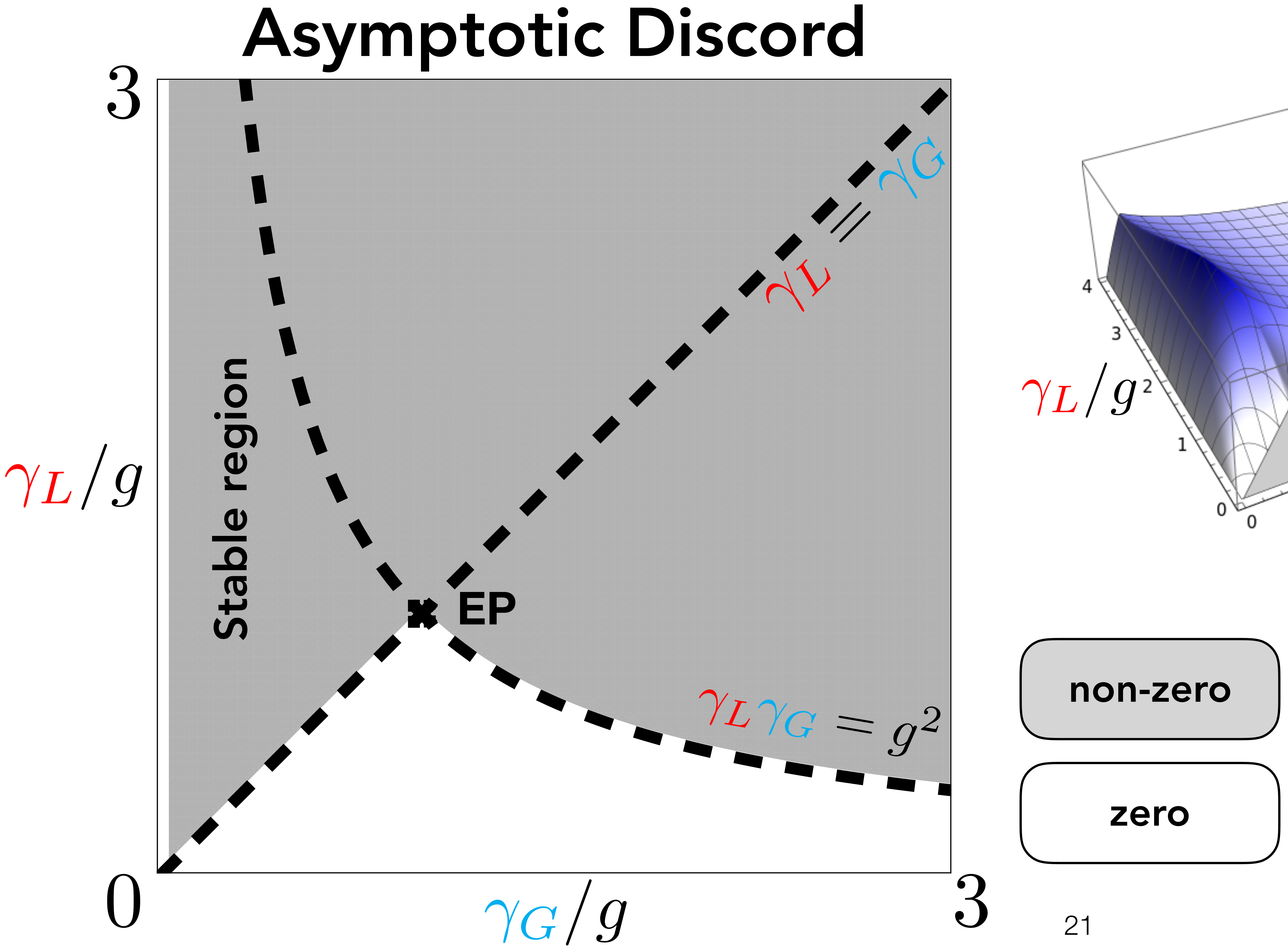
Dynamics of quantum correlations

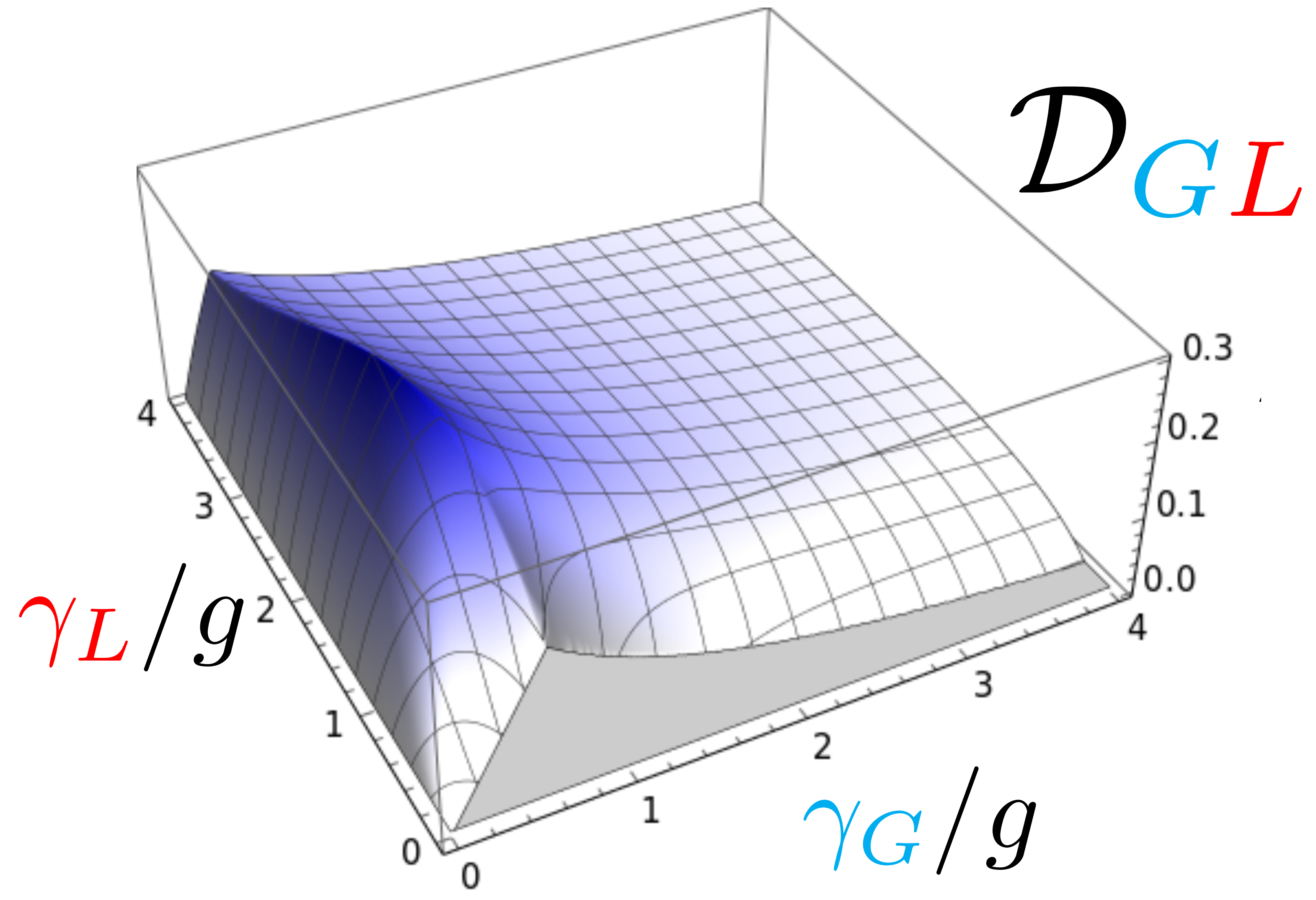


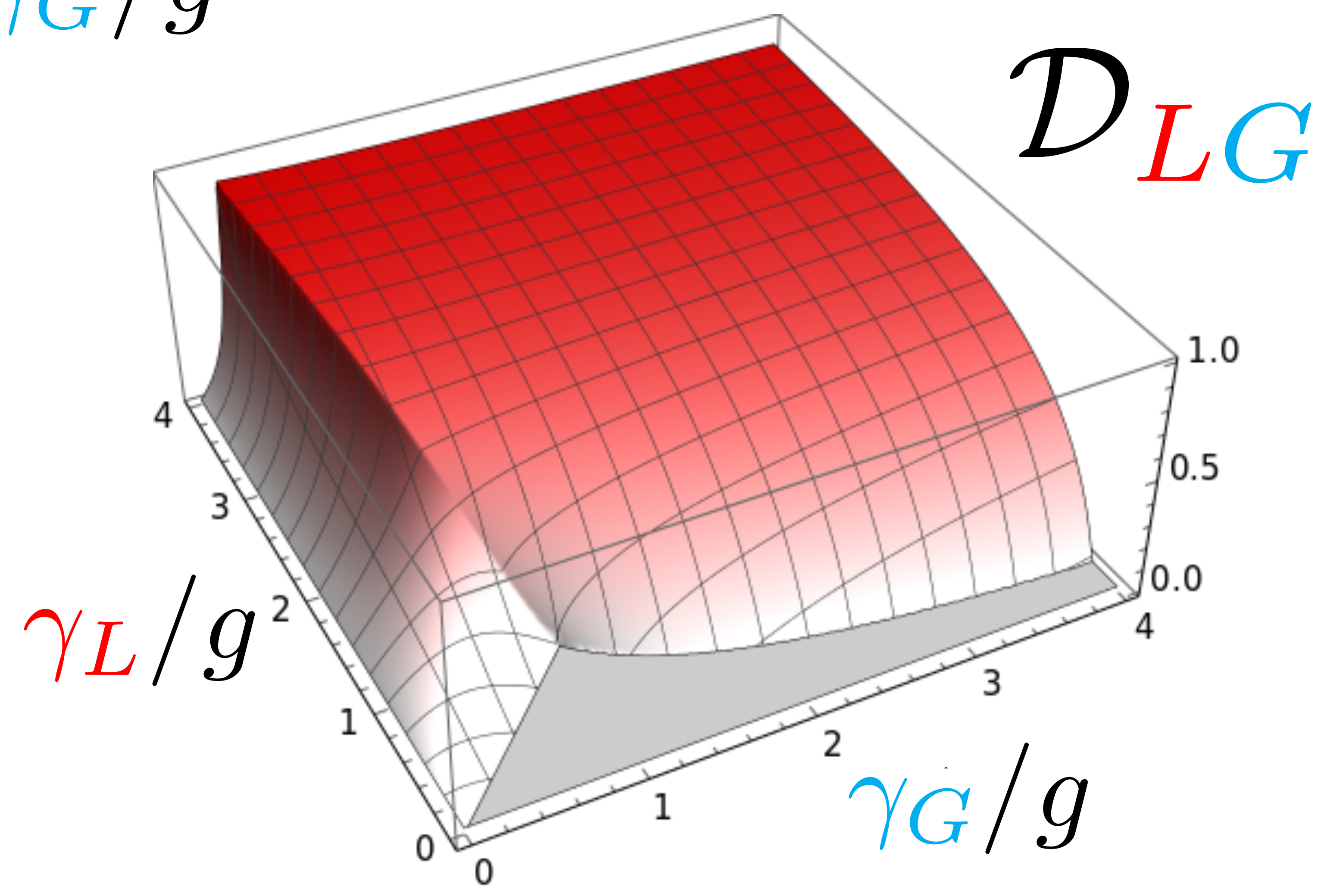


critical exponent
$1/2$

Dynamics of quantum correlations







Physical interpretation

Discord Generation (~clear..)

Loss

$$|\alpha\rangle \rightarrow |\eta\alpha\rangle\,, \quad \eta < 1$$

Purity unaffected

S. Haroche and J.-M. Raimond, Exploring the Quantum: Atoms, Cavities, and Photons (Oxford University Press, 2006).

Coupling

$$|\alpha\rangle\langle\alpha| \otimes |\beta\rangle\langle\beta|$$
$$\downarrow$$
$$|\tilde{\alpha}\rangle\langle\tilde{\alpha}| \otimes |\tilde{\beta}\rangle\langle\tilde{\beta}|$$

Beam splitter

S. Haroche and J.-M. Raimond, Exploring the Quantum: Atoms, Cavities, and Photons (Oxford University Press, 2006).

Gain

$$|\alpha\rangle\langle\alpha|$$
$$\downarrow$$
$$\int d^2\alpha' P(\alpha')\,|\alpha'\rangle\langle\alpha'|$$

- Purity diminished
- Superposition of non orthogonal states

S. Scheel and A. Szameit, Euro Phys. Lett. 122, 34001 (2018).
N. Korolkova and G. Leuchs, Rep. Prog. Phys. 82, 056001 (2019).

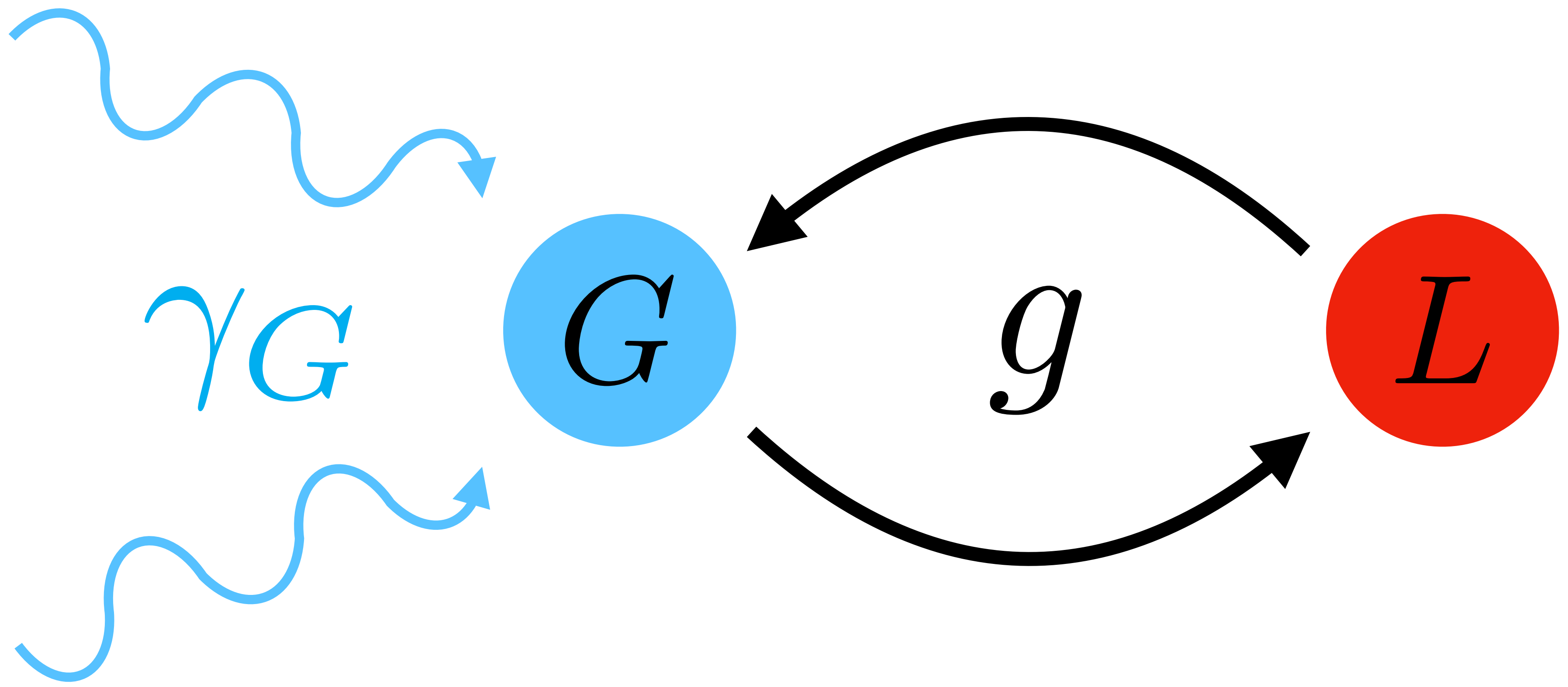
Physical interpretation

<u>Discord Generation</u> (~clear..)

$$\rho_0 = |\alpha_G\rangle\langle\alpha_G| \otimes |\alpha_L\rangle\langle\alpha_L|$$

$$\int d^2\tilde{\alpha}'_G P(\tilde{\alpha}'_G)\, |\tilde{\alpha}'_G\rangle\langle\tilde{\alpha}'_G| \otimes |\tilde{\alpha}'_L\rangle\langle\tilde{\alpha}'_L|$$

N. Korolkova and G. Leuchs, Rep. Prog. Phys. 82, 056001 (2019).



Physical interpretation

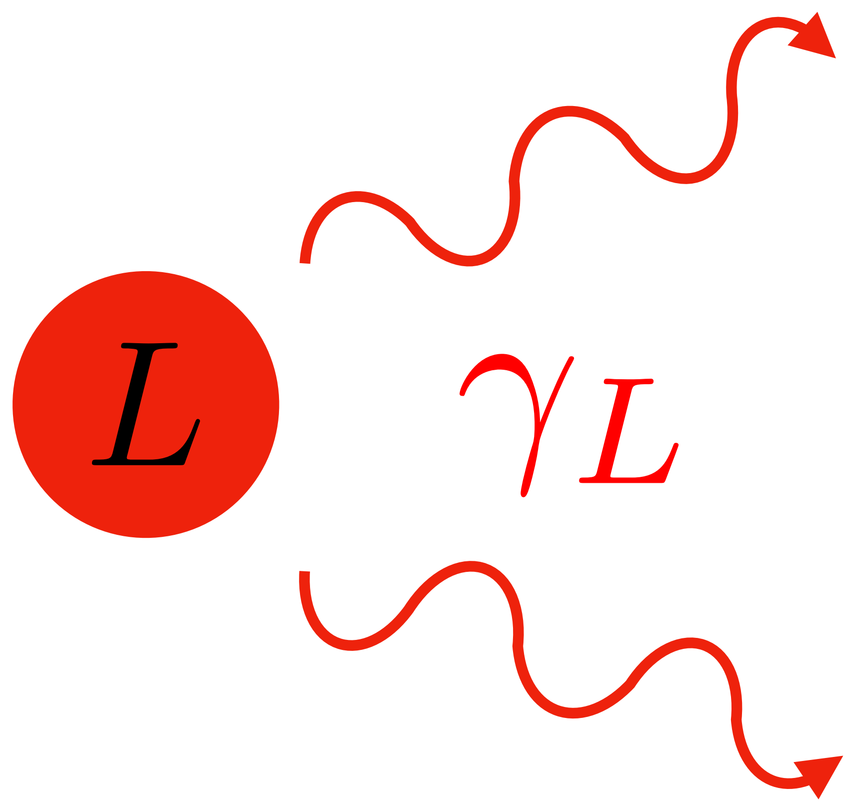
Discord Generation (~clear..) … & stabilisation (~?)

$$\rho_0 = |\alpha_G\rangle\langle\alpha_G| \otimes |\alpha_L\rangle\langle\alpha_L|$$

$$\int d^2\tilde{\alpha}'_G P(\tilde{\alpha}'_G)\, |\tilde{\alpha}'_G\rangle\langle\tilde{\alpha}'_G| \otimes |\tilde{\alpha}'_L\rangle\langle\tilde{\alpha}'_L|$$

N. Korolkova and G. Leuchs, Rep. Prog. Phys. 82, 056001 (2019).

$$\int d^2\tilde{\alpha}'_G P(\tilde{\alpha}'_G)\, |\tilde{\alpha}'_G\rangle\langle\tilde{\alpha}'_G| \otimes |\eta\tilde{\alpha}'_L\rangle\langle\eta\tilde{\alpha}'_L|$$



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N. Korolkova and G. Leuchs, Rep. Prog. Phys. 82, 056001 (2019).

$$\int d^2\tilde{\alpha}'_G P(\tilde{\alpha}'_G)\,|\tilde{\alpha}'_G\rangle\langle\tilde{\alpha}'_G| \otimes |\eta\tilde{\alpha}'_L\rangle\langle\eta\tilde{\alpha}'_L|$$

In a $L - L$ or $G - G$ system

~~Stable Quantum Correlations~~

Stable Quantum Correlations

Highlights

- **Quantum** properties of **PT symmetric** system
- For equal **gain** and **loss** rates **Quantum Correlations** (**QCs**) **decay** in the **exact** phase and are **finite** in the **broken** phase
- **Gain: creation** of **QCs.** **Gain/Loss: stabilisation** of **QCs**
- Useful for Quantum Technologies G. Adesso, T. R. Bromley, and M. Cianciaruso, J. Phys. A: Math. Theor. 49, 473001 (2016).
  - information encoding M. Gu, H. M. Chrzanowski, S. M. Assad, T. Symul, K. Modi, T. C. Ralph, V. Vedral, and P. K.Lam, Nat. Phys. 8, 671 (2012).
  - remote state preparation B. Dakic, Y. O. Lipp, X. Ma, M. Ringbauer, S. Kropatschek, S. Barz, T. Paterek, V. Vedral, A. Zeilinger, C. Brukner, and P. Walther, Nat. Phys. 8, 666 (2012).
  - entanglement activation M. Piani, S. Gharibian, G. Adesso, J. Calsamiglia, P. Horodecki, and A. Winter, Phys. Rev. Lett. 106, 220403 (2011).
  - entanglement distribution C. E. Vollmer, D. Schulze, T. Eberle, V. Haendchen, J.Fiurasek, and R.Schnabel, Phys.Rev.Lett.111, 230505 (2013).

Thank you!


Noise?
Entanglement?
E
r
λ
$|\psi|^2$
x
?
$|\psi|^2$
x
$E_n - E_m = \frac{Z^2 e^4 m}{2\hbar^2}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$