# Quench action & large deviations: work statistics in the 1d Bose gas


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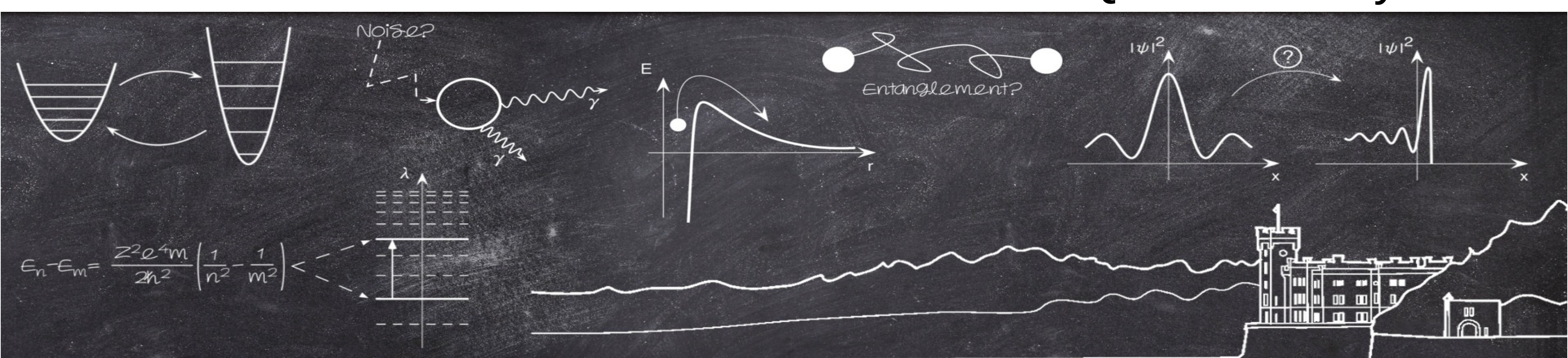
Trieste Junior Quantum Days 2019



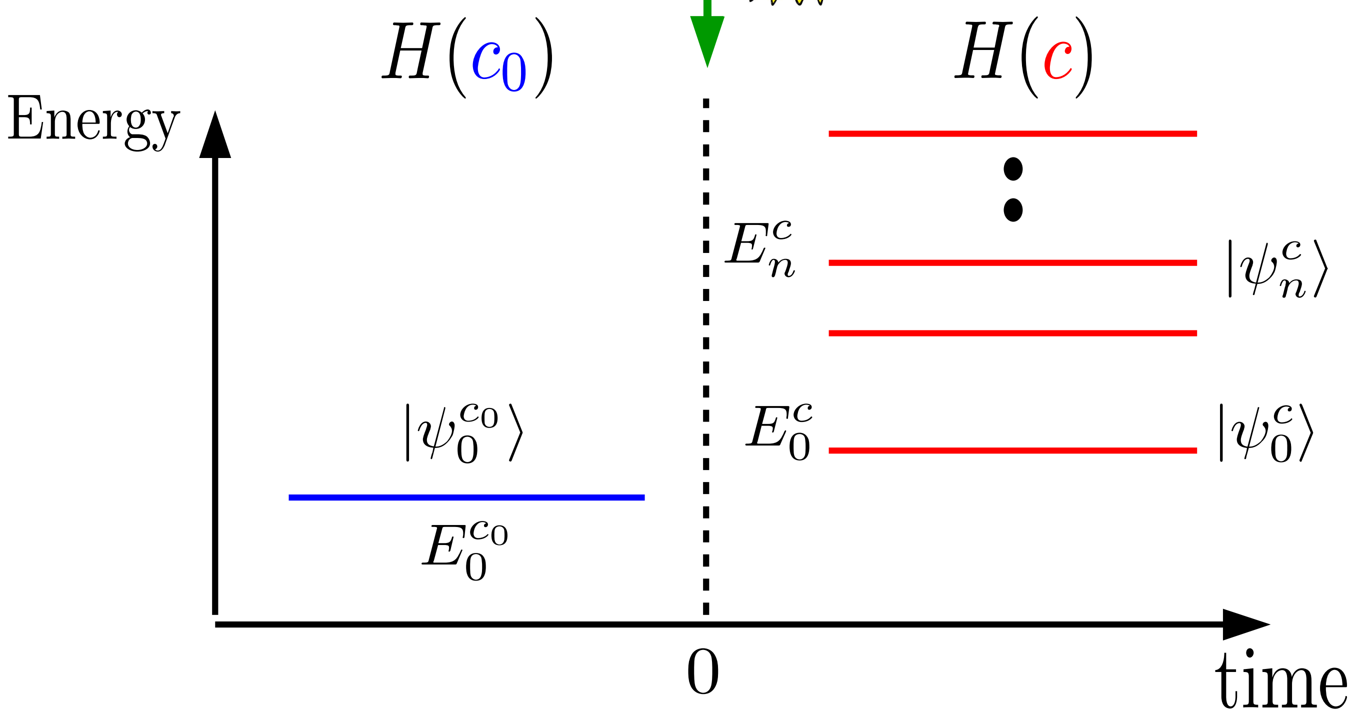
# Quantum quenches

*Global* quantum quench

Calabrese, Cardy 2006 +

...

$$\left\{ \begin{array}{l} H(c_0) \\ |\psi_0^{c_0}\rangle \end{array} \right. \longrightarrow \begin{array}{l} H(c) \\ |\psi(t)\rangle = \exp(-iH(c)t)\,|\psi_0^{c_0}\rangle \end{array}$$


- Dynamics,
- correlations,
- entanglement
- thermalization

Here *fluctuating* quantities

↓

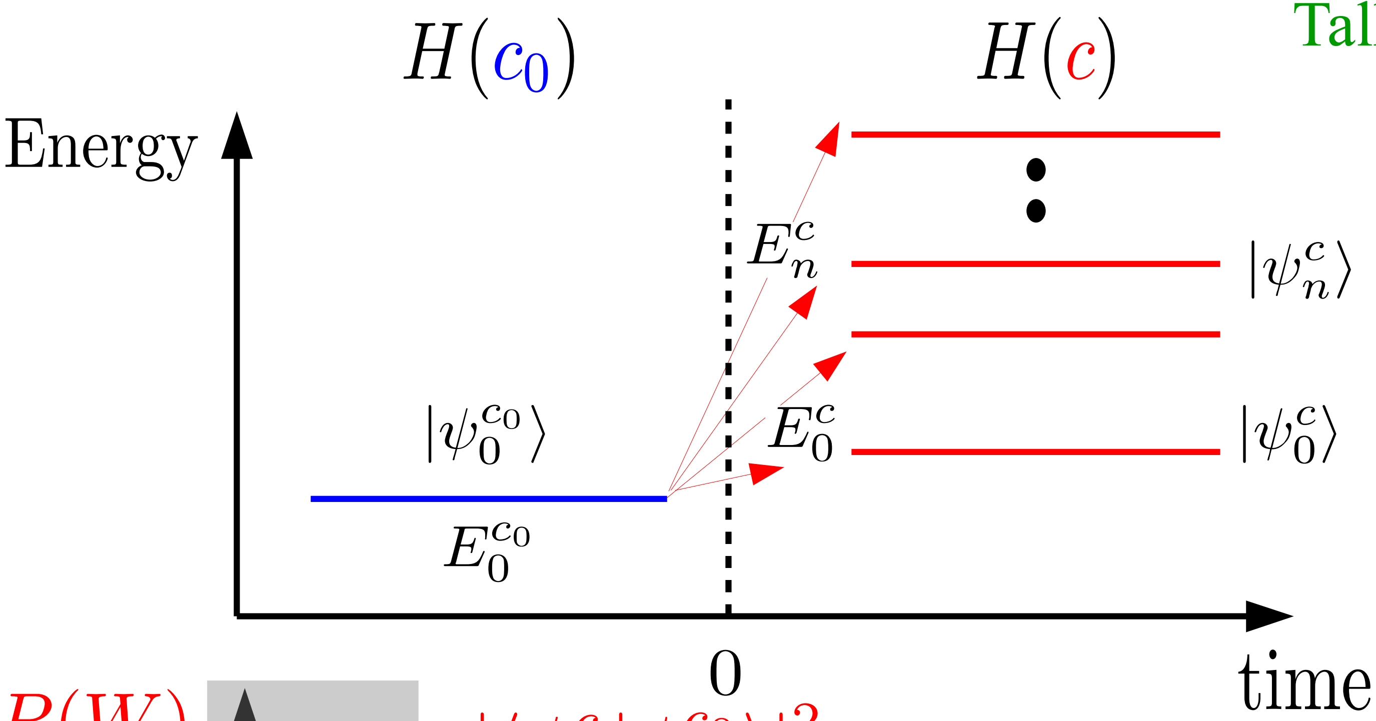
Work

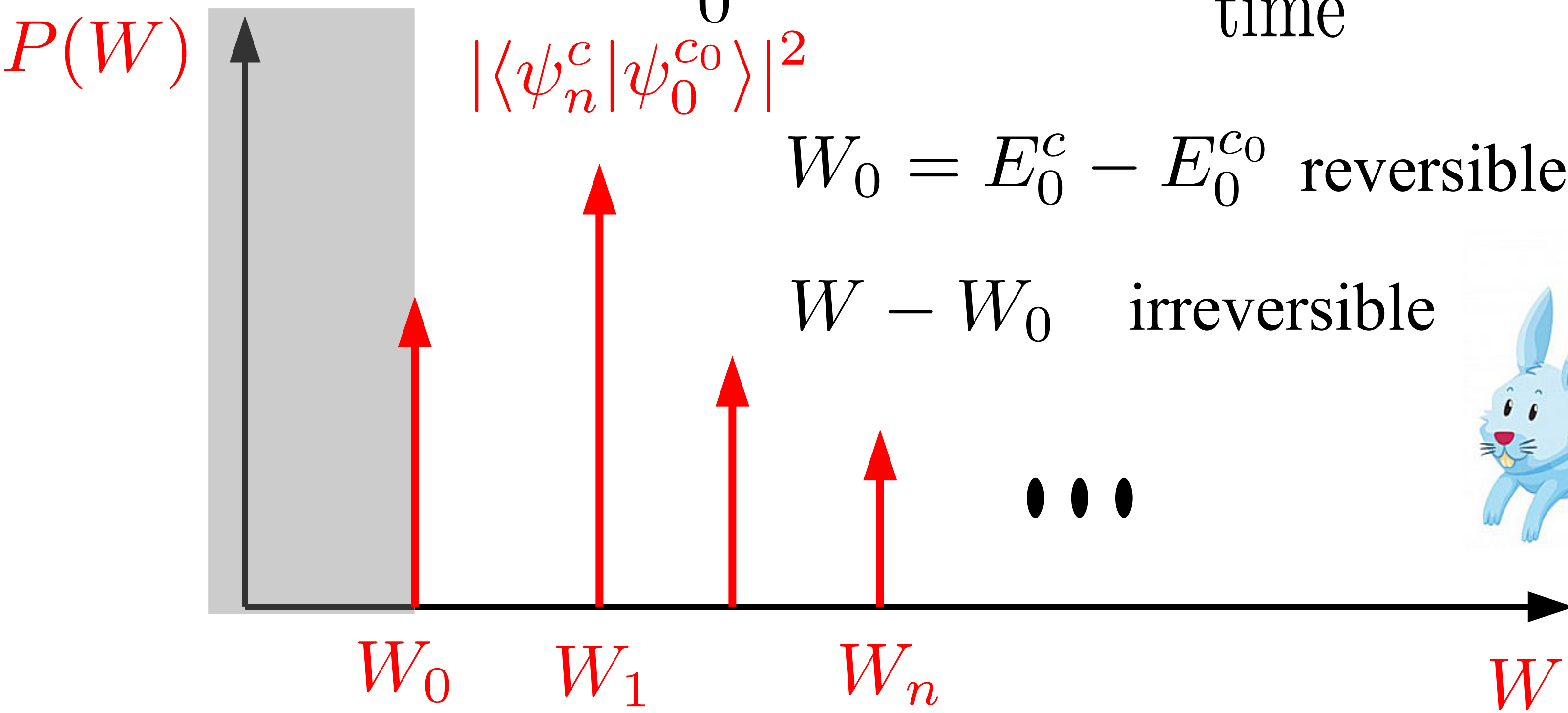
$$W$$

# Work statistics

Talkner, Lutz, Hänggi 2007

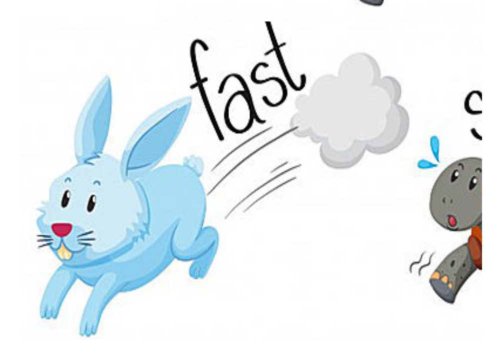
$$W = E_n^c - E_0^{c_0}$$

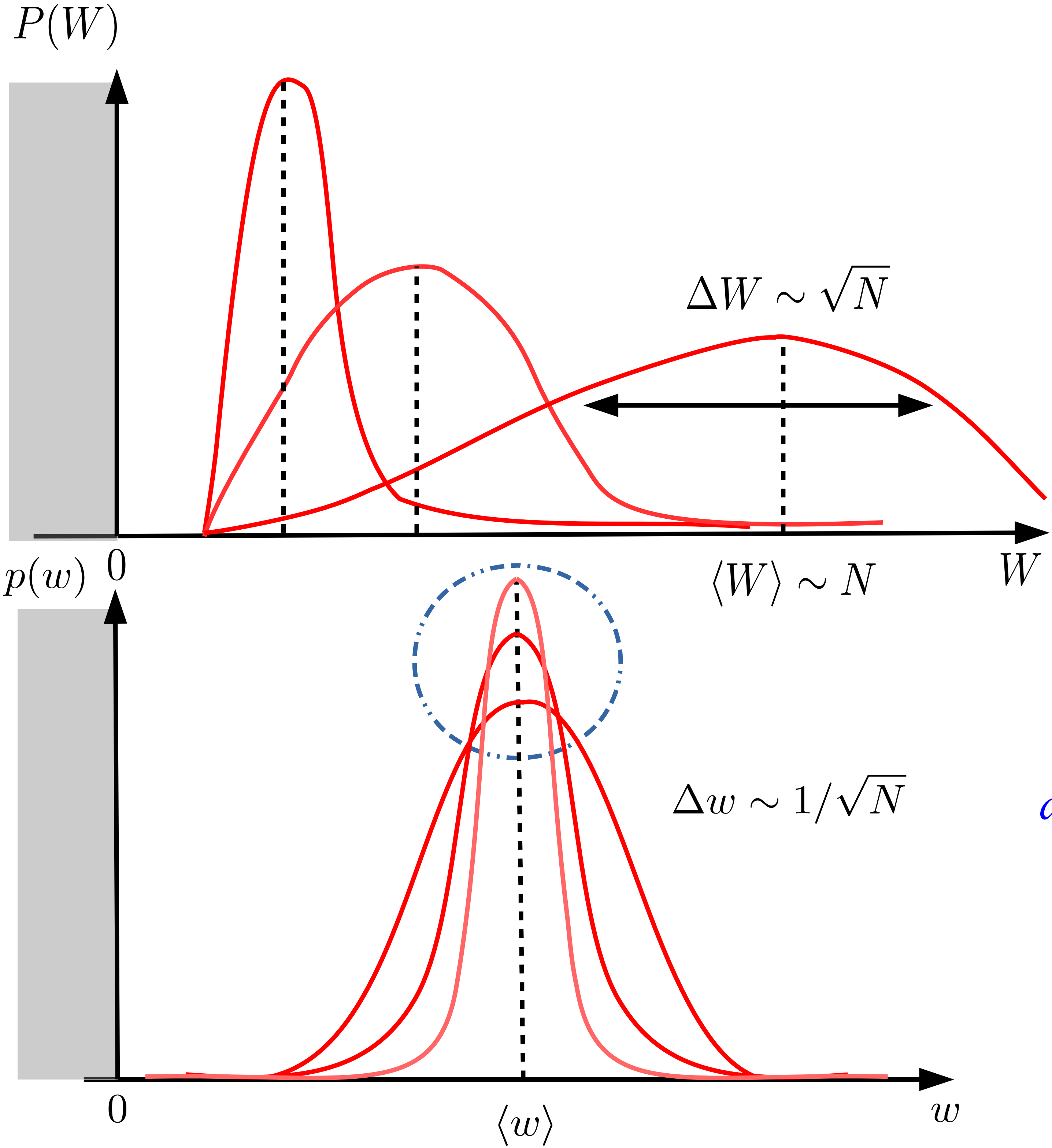



$W_0 = E_0^c - E_0^{c_0}$ reversible

$W - W_0$ irreversible




$$\frac{\Delta W}{W} \sim \frac{1}{\sqrt{N}}$$

$$w \equiv W/N$$

*Gaussian distribution*

↓

Large deviations ?

Touchette 2009

# Large deviations

$$G(s) = \langle \mathrm{e}^{-sW} \rangle = \langle \psi_0^{c_0} | \mathrm{e}^{-s[H-E_0^c]} | \psi_0^{c_0} \rangle$$

$$= e^{-Nf(s)}$$

SCGF

$$p(w) \propto \int \mathrm{d}s\, \mathrm{e}^{N(s\,w - f(s))}$$

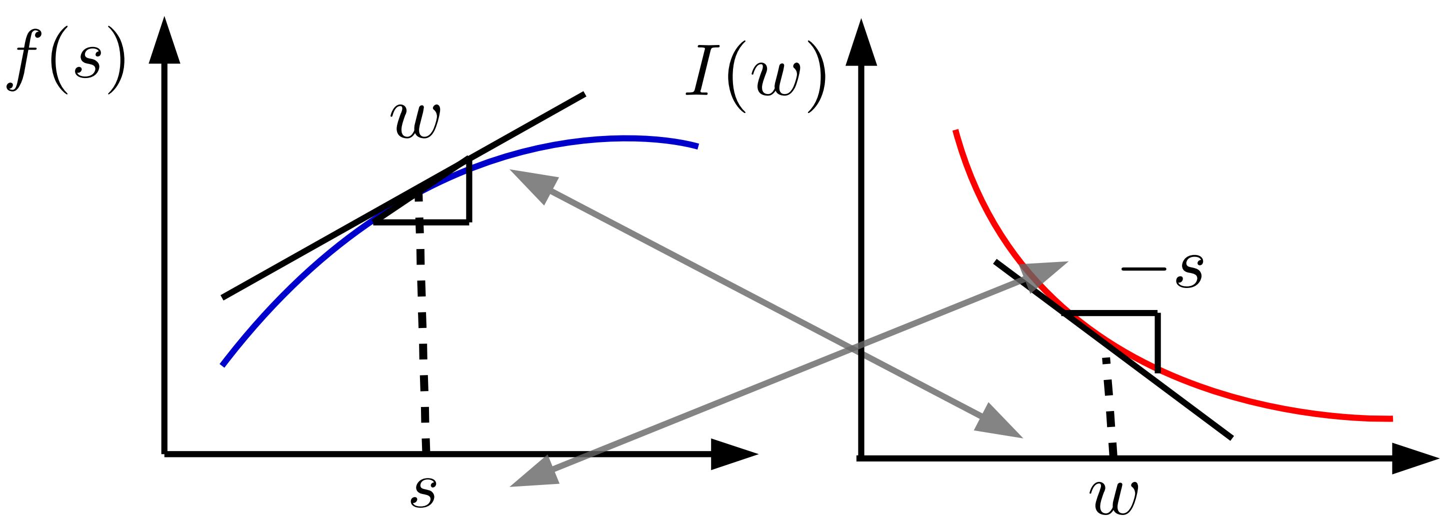
$$p(w) \propto \mathrm{e}^{-NI(w)}$$

*Large deviation principle* $I(\bar{w}) = 0$

$$I(w) = -\inf_s\{sw - f(s)\}$$

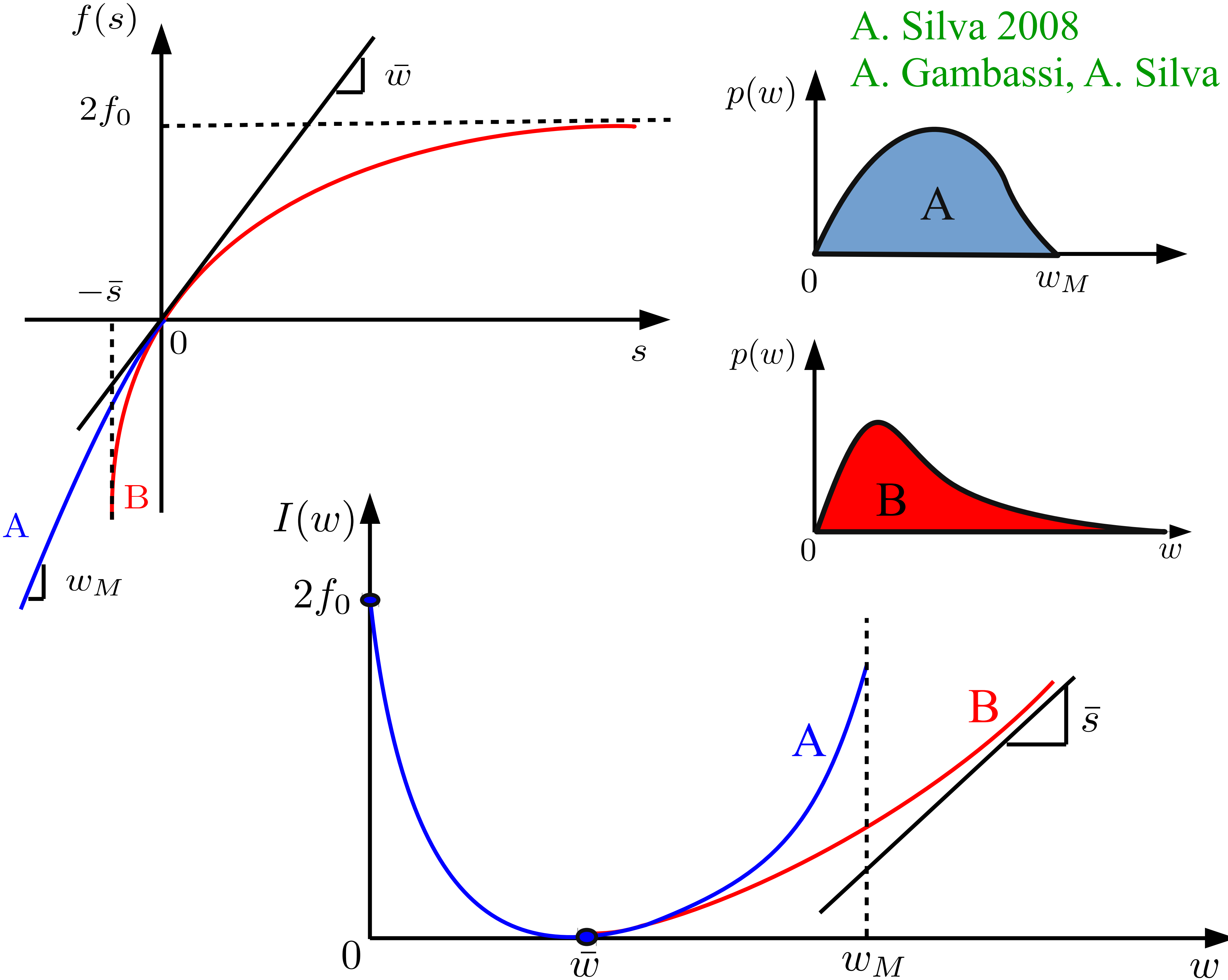
*Large deviation function*

$$\left\{ \begin{aligned} \frac{\mathrm{d}I(w)}{\mathrm{d}w} &= -s \\ \frac{\mathrm{d}f(s)}{\mathrm{d}s} &= w \end{aligned} \right.$$



A. Silva 2008

A. Gambassi, A. Silva 2012



# Free bosonic field

Sotiriadis, Gambassi, Silva 2013

$$H(m) = \frac{1}{2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\left[\pi_\phi^2 + (\partial_x \phi)^2 + m^2\phi^2\right]$$

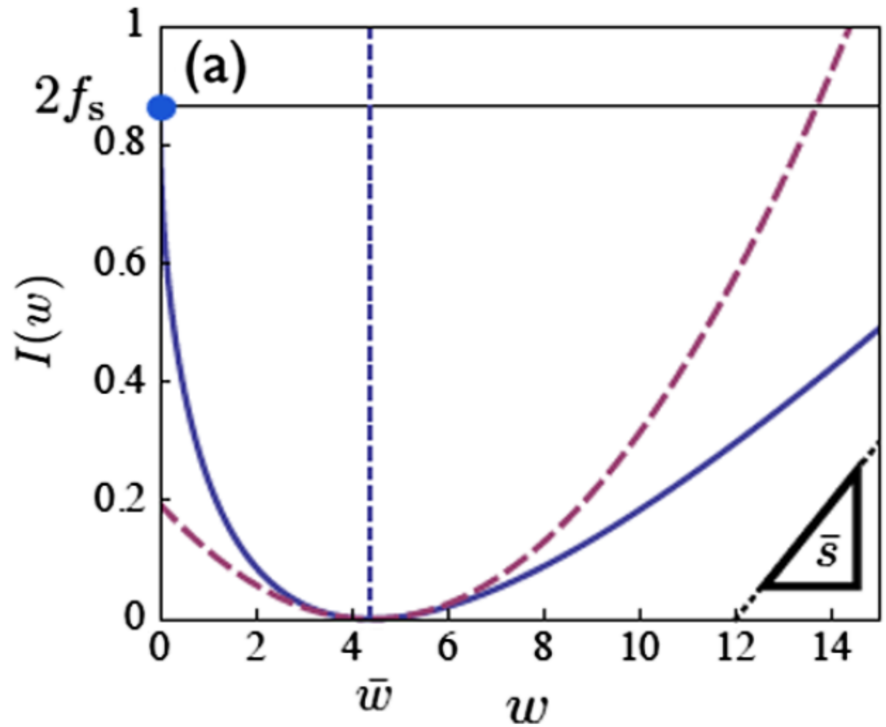
$$\omega(k) = \sqrt{k^2 + m^2} \longrightarrow \text{quench} \quad \lambda_k(m_0, m)$$

$$f(s) = \frac{1}{2}\int \frac{\mathrm{d}^d k}{(2\pi)^d} \ln\left[\frac{1-\lambda_k^2\,\mathrm{e}^{-2\omega_k\, s}}{1-\lambda_k^2}\right]$$

Class B $s > -\bar{s}$

$$\downarrow$$

$$I(w)$$



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Class B $s > -\bar{s} = 0$

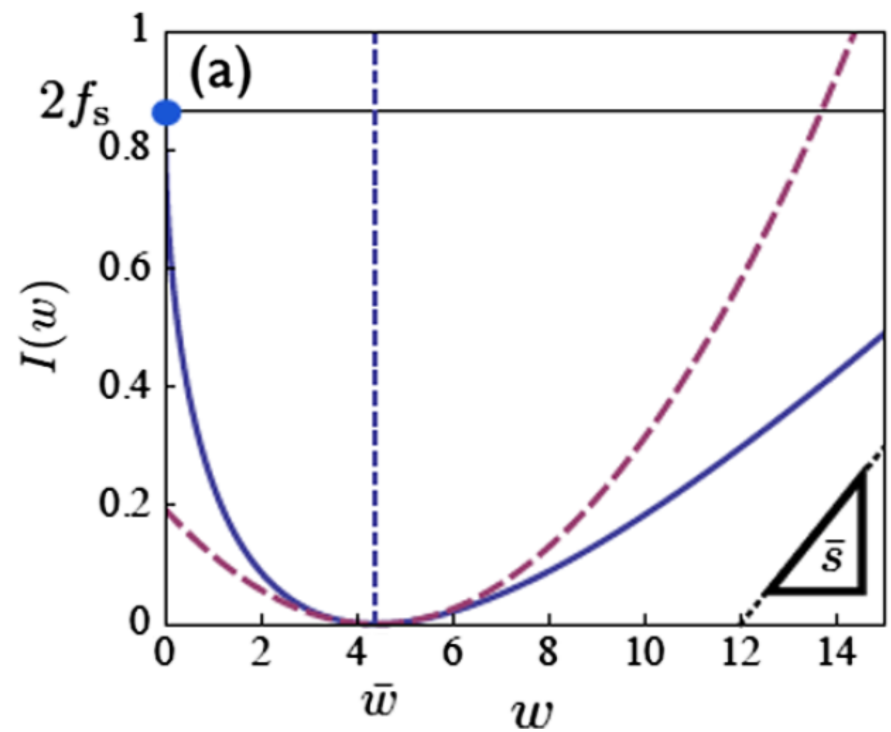
$$\downarrow$$

$$I(w)$$

$$m_0 = 0$$

$$I(w) \equiv 0 \quad \text{for} \quad w > \bar{w}$$

*Condensation*



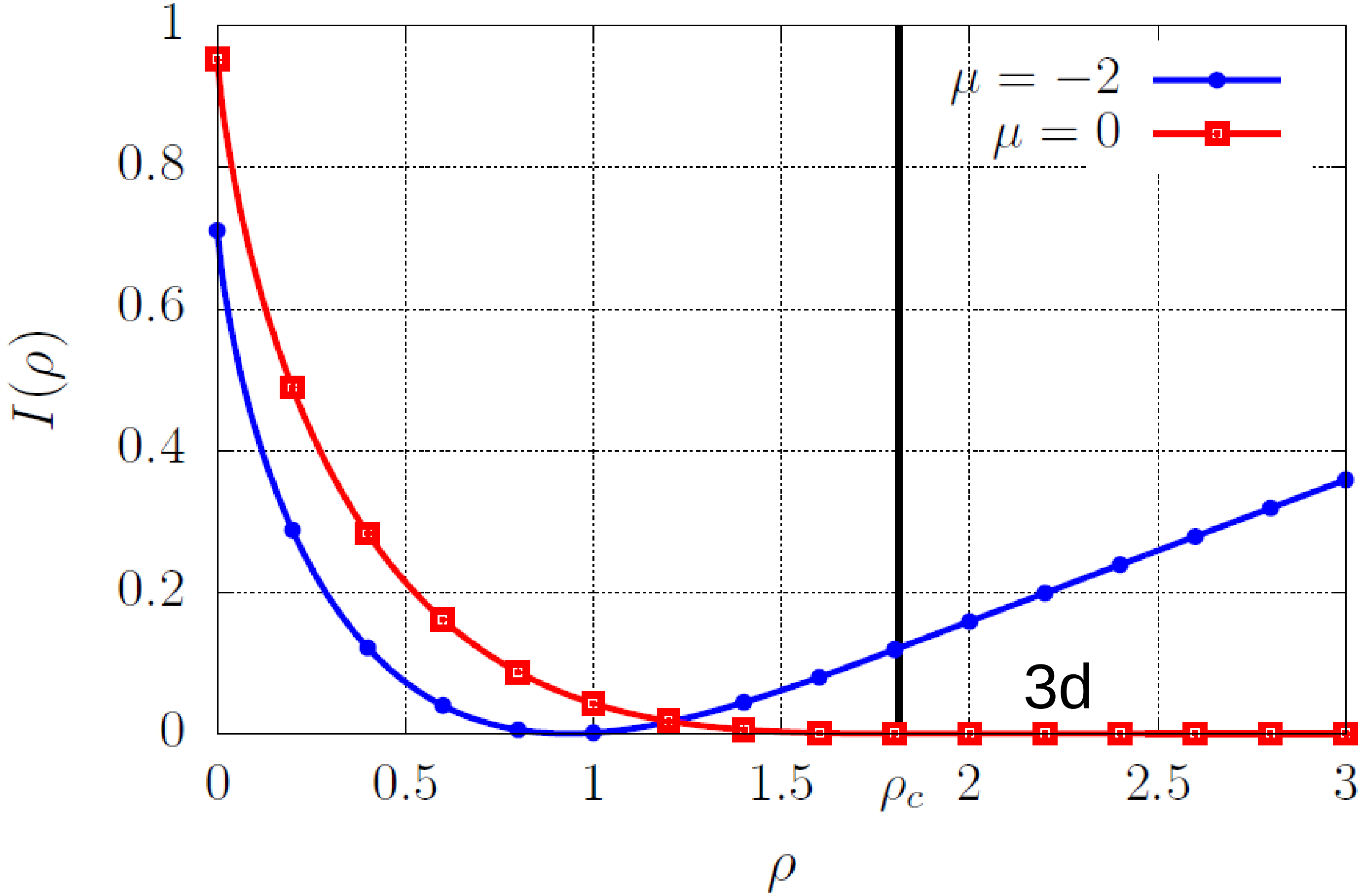
# Condensation

$$\begin{cases} f(s) \longleftrightarrow \psi(s) \\ m_0 \longleftrightarrow \mu \end{cases}$$

Bose gas $\mu \leq 0$

BEC at $\mu = 0$

$d > 2$

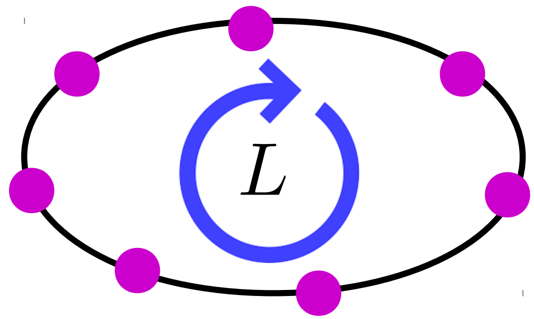



$$\rho_c = l^{-3}\xi(3/2)$$

$$l = \sqrt{\frac{2\pi\beta\hbar^2}{m}}$$

Aim: $\begin{cases} \text{Quantitative description of interacting systems?} \\ \text{Condensed regime?} \end{cases}$

# Lieb Liniger model




$$H(c) = -\frac{\hbar^2}{2m}\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2} + 2c\sum_{j<k}\delta(x_j - x_k)$$

$$\hbar^2 = 2m = 1$$

$$H(c) = \int_0^L \mathrm{d}x\, \partial_x \Psi^\dagger \partial_x \Psi + c\, \Psi^\dagger \Psi^\dagger \Psi \Psi$$

$$\gamma = \frac{c}{D}$$

$$\left[\Psi(x), \Psi^\dagger(x')\right] = \delta(x - x')$$

$$N, L \to \infty \qquad D = \frac{N}{L}$$

GS of $H(c = 0)$

$W$

$$|\mathrm{BEC}\rangle = \frac{1}{\sqrt{N!}}\int_0^L \mathrm{d}^N\mathrm{x}\; \psi(x_1, x_2 \ldots x_N)\; \Psi^\dagger(x_1)\Psi^\dagger(x_2)\ldots\Psi^\dagger(x_N)\;|0\rangle$$

$$\psi(x_1, x_2 \ldots x_N) = \frac{1}{L^{N/2}}$$

$H(c)$ $c > 0$ repulsive

# Work statistics & quench action

$$G(s) = \langle \psi_0^{c_0} | \mathrm{e}^{-s[H - E_0^c]} | \psi_0^{c_0} \rangle$$

$$= \sum_{\{\lambda_j\}} |\langle \{\lambda_j\} | \mathrm{BEC} \rangle|^2 \mathrm{e}^{-s(E[\{\lambda_j\}] - E_0^c)}$$

Bethe ansatz

$$H_c | \{\lambda_j\} \rangle = E | \{\lambda_j\} \rangle$$

Thermodynamic limit

$$\sum_{\{\lambda_j\}} \longrightarrow \int \mathcal{D}\rho \, \mathrm{e}^{L \, S_{\mathrm{yy}}[\rho]}$$

$$|\{\lambda_j\}\rangle \rightarrow \rho(\lambda), \rho^h(\lambda)$$

J.-S. Caux and F. H. L. Essler 2013

J.-S. Caux 2016

$$|\langle \{\lambda_j\} | \mathrm{BEC} \rangle|^2 = e^{-L \, 2 S_O[\rho]}$$

J. De Nardis, B. Wouters, M. Brockmann, J.-S. Caux, 2014

$$= \int \mathcal{D}\rho \, \exp[-L(s(e[\rho] - e_0(c)) - \frac{1}{2} S_{\mathrm{YY}}[\rho] + 2 S_O[\rho] + S_N[\rho])]$$

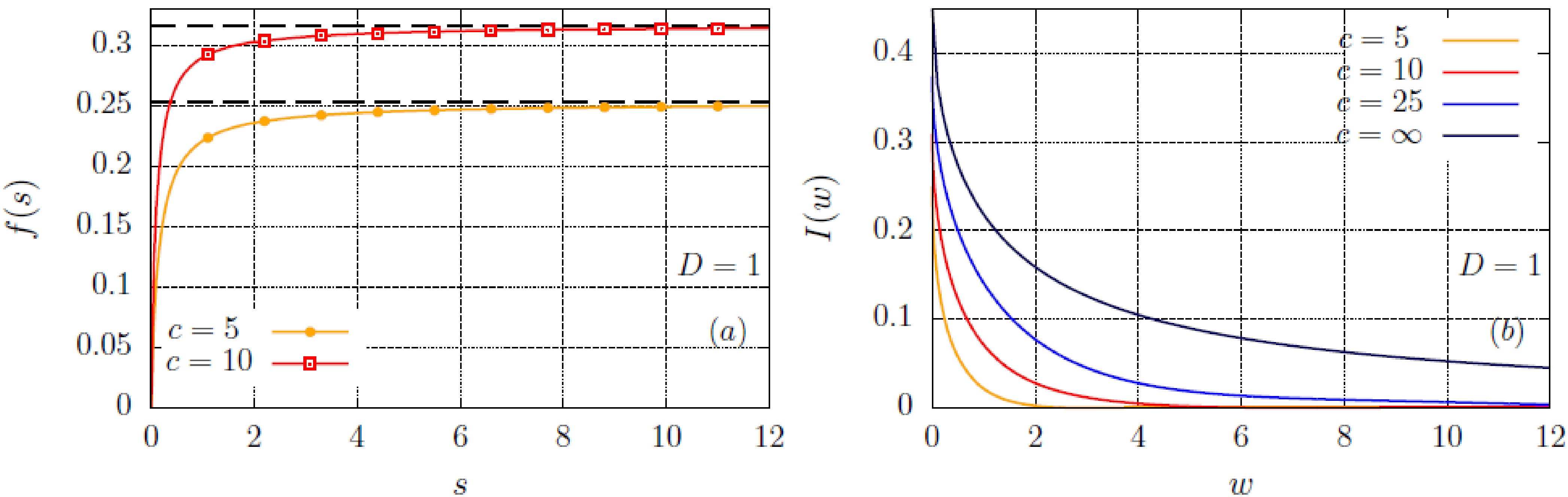
$$\rho \rightarrow \rho_s^*$$

$$S_{\mathrm{QA}}[s, \rho_s]$$

$$\simeq \exp[-L(S_{\mathrm{QA}}[s, \rho_s^*] - s e_0(c))]$$

# Results

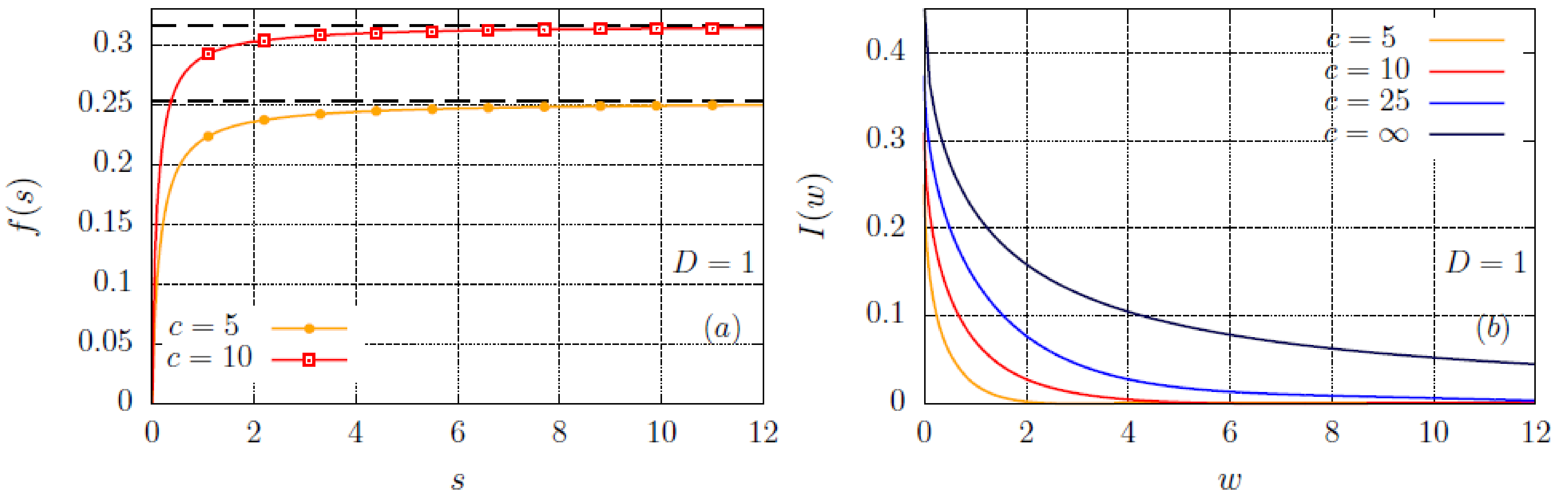
Class B $s \geq 0$




$$\bar{w} = \frac{1}{L} \langle \mathrm{BEC}|H(c) - E_0(c)|\mathrm{BEC}\rangle = cD^2 - e_0(c)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2} f(s)\Big|_{s=0} = \langle \mathrm{BEC}| \left[H(c) - E_0(c)\right]^2 |\mathrm{BEC}\rangle \to \infty$$ → Non analytic behavior in the origin

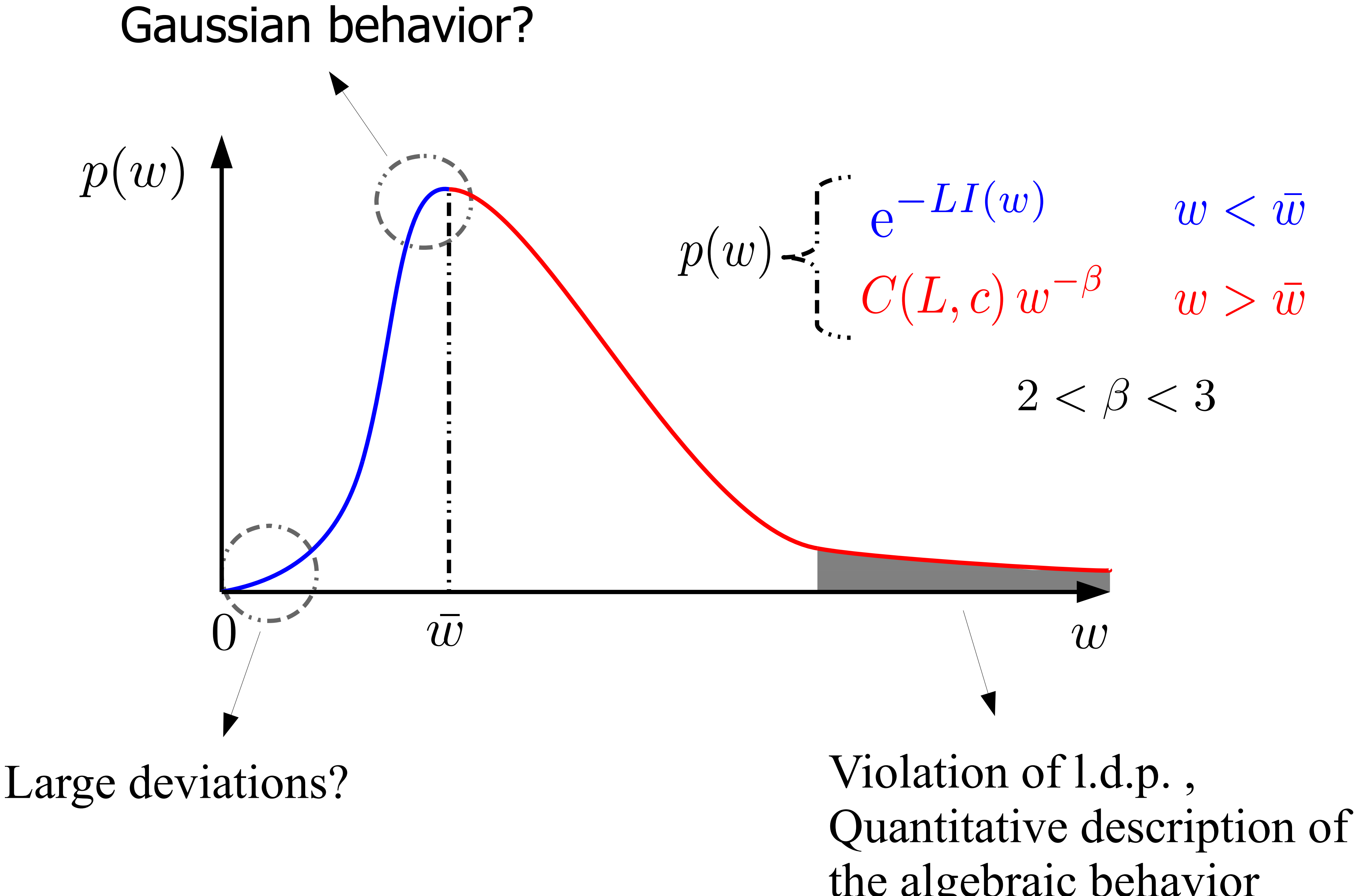
# Results




$I(\bar{w}) = 0$ Mean intensive work

$I(w) \equiv 0 \quad w > \bar{w}$ → *Condensation*

*Algebraic behavior*


Gaussian behavior?
$p(w)$
$p(w)$
$e^{-LI(w)}$ $w < \bar{w}$
$C(L,c)\, w^{-\beta}$ $w > \bar{w}$
$2 < \beta < 3$
0
$\bar{w}$
$w$
Large deviations?
Violation of l.d.p. ,
Quantitative description of
the algebraic behavior

# Small and large deviations

$s \to 0$

$$f(s) = [cD^2 - e_0(c)]s - \frac{2}{3}c^2D^2\sqrt{\frac{2}{\pi}}s^{3/2} + O(s^2)$$

**Not** Gaussian !

$$I(w \to \bar{w}^-) = \frac{\pi}{6c^4D^4}(\bar{w} - w)^3 + O((\bar{w} - w)^4)$$ for $w \leq \bar{w}$

$s \to \infty$

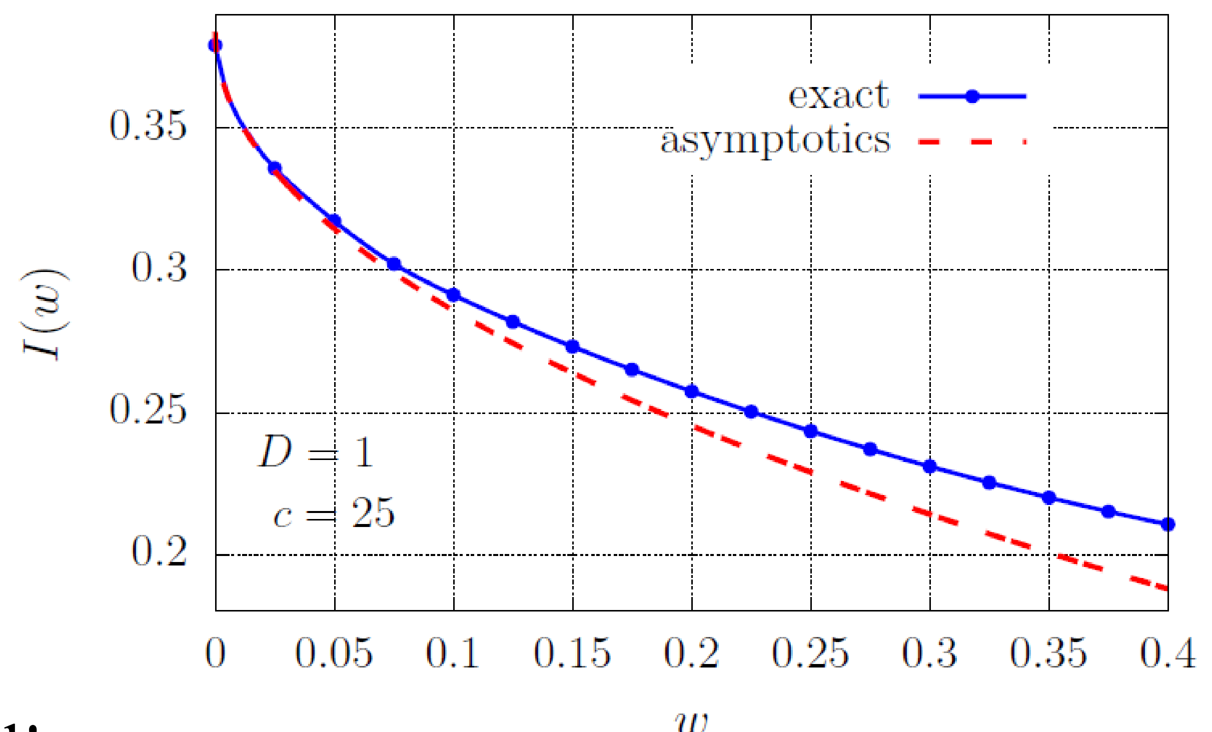
$$f(s) = 2f_0 + \frac{f_1}{s} + O(s^{-2})$$

↕

$$I(w) - 2f_0 = -2\sqrt{-f_1}w^{1/2}$$

$\propto w^{d/(d+1)}$

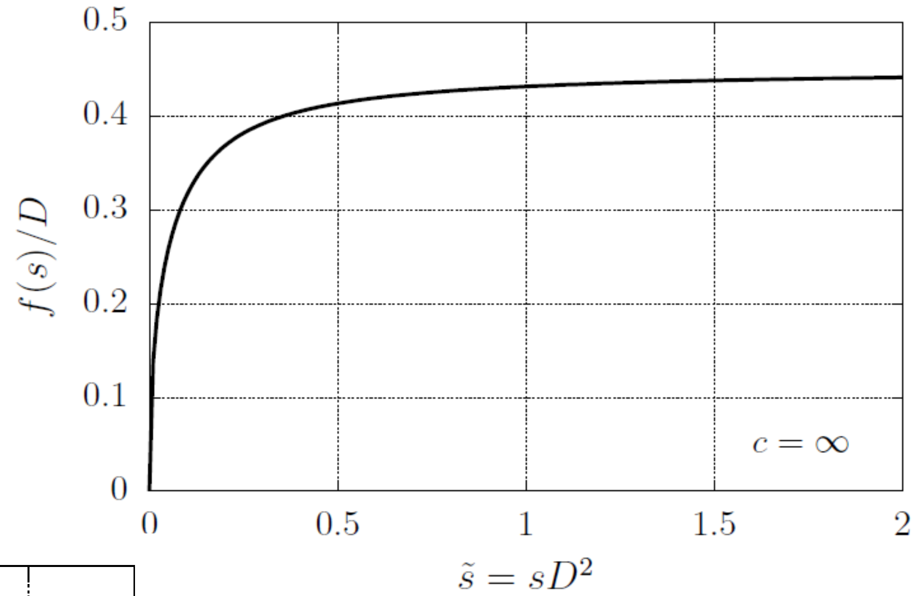
Universal!

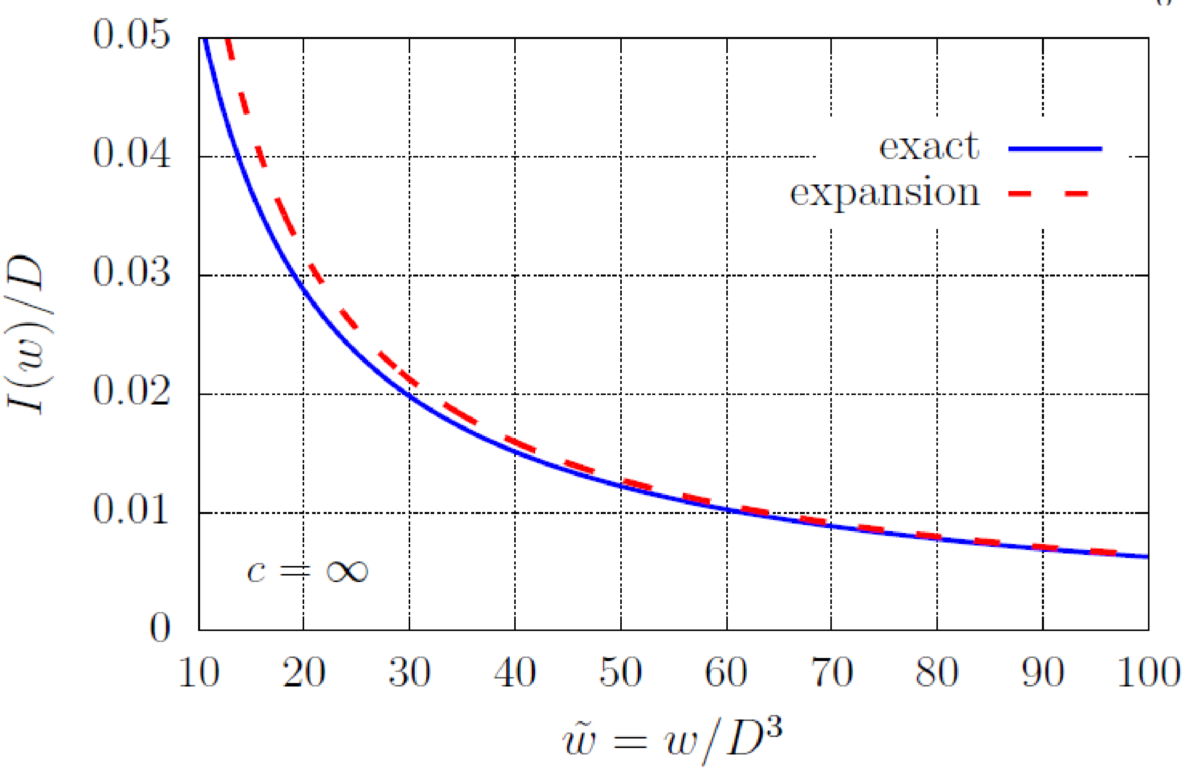


# Tonks – Girardeau limit

$$c \to \infty$$

$$\frac{f(s)}{D} = \alpha_{1/2}\, \tilde{s}^{1/2} + O(\tilde{s})$$




$$\frac{I(w)}{D} = \frac{\alpha_{1/2}^2}{4}\tilde{w}^{-1} + O\left(\tilde{w}^{-2}\right)$$

No algebraic behavior

# Algebraic behaviour

$$I(w) \equiv 0 \qquad w > \bar{w} \qquad\qquad p(w) \sim C(L,c)\, w^{-\beta}$$

Low density $D = \dfrac{N}{L} \to 0, \quad L \to \infty$

$$\begin{cases} \lambda_j = \dfrac{2\pi I_j}{L} + O(L^{-2}) \\ \langle \{\lambda_j\} | \mathrm{BEC} \rangle \simeq \dfrac{\sqrt{(cL)^{-M}}\sqrt{N!}}{\prod_{j=1}^{M} \left( \frac{\lambda_j}{c}\sqrt{\frac{1}{4} + \frac{\lambda_j^2}{c^2}} \right)} \end{cases} \qquad N = 2M+1$$

↓

$$p(w) = L\, P(wL)$$

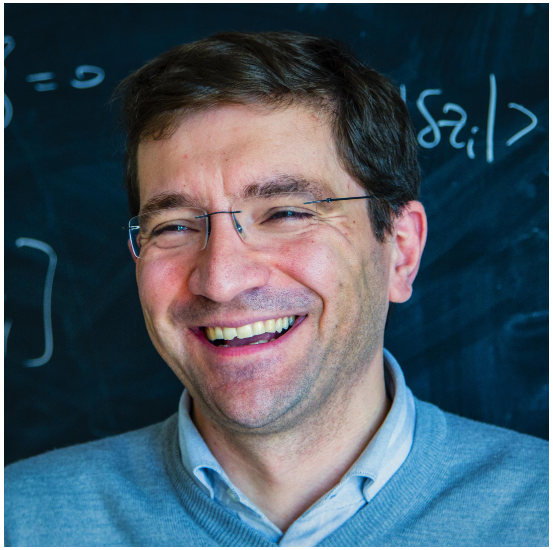
$$\boxed{p(w) \propto \frac{c^2}{L^{5/2}}\, w^{-5/2}}$$

# Conclusions

- Work statistics in interacting integrable model (L.L.)
- Interaction quench from a BEC initial state
- Quench action method for $w < \bar{w}$
- Non Gaussian behavior of small fluctuations
- Large deviations exhibit universal properties
- Violation of large deviation principle for $w > \bar{w}$
- Algebraic behavior and condensation, low density limit

# Thanks to

L. Piroli



A. Gambassi

*And to you for the attention*