Near Field Interferometry with Large Particles

Alessio Belenchia

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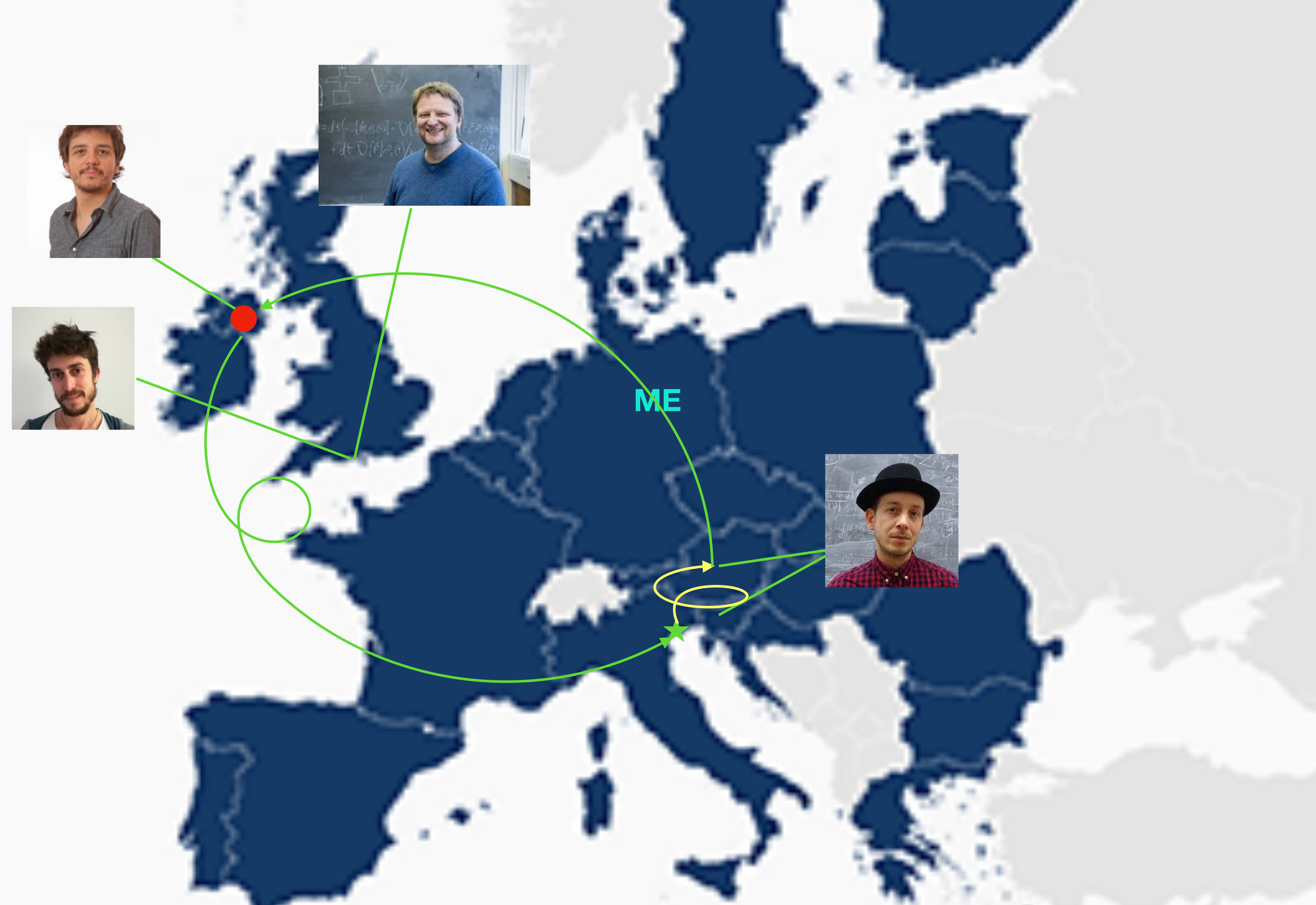
Trieste Junior Quantum Days

ICTP, Trieste

Trieste, 24-26 July 2019



AB, G. Gasbarri, R. Katelbeck, H. Ulbricth and M. Paternostro, arXiv.1907.04127

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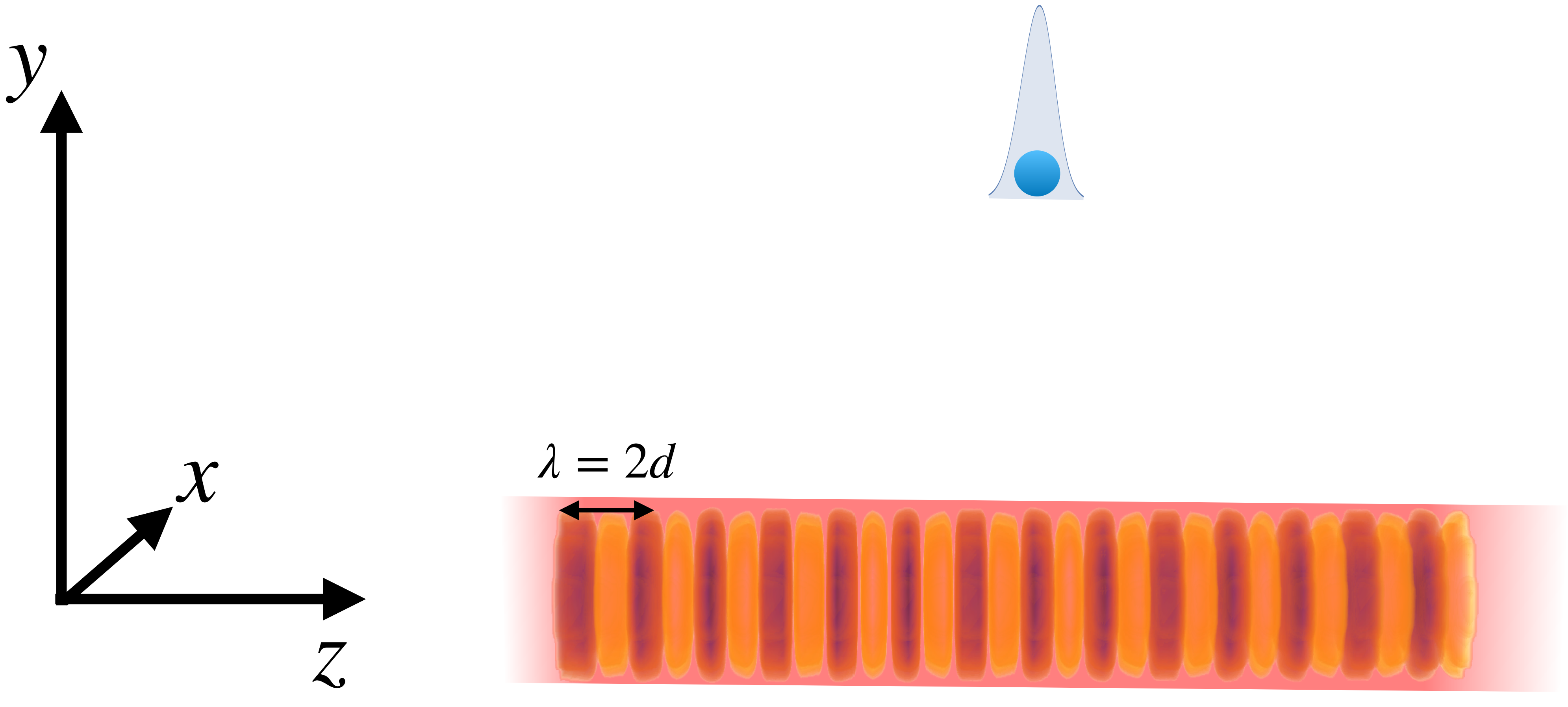
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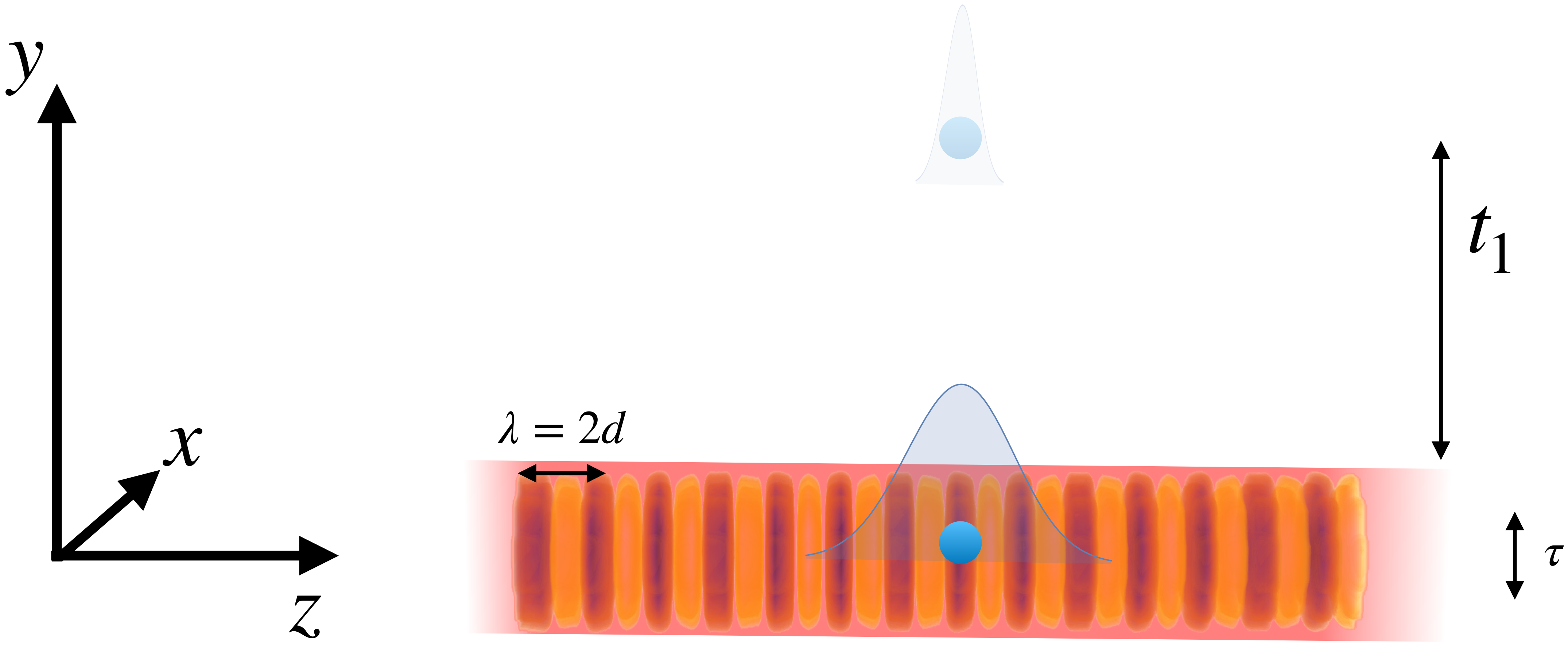
λ = 2d


y
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$\lambda = 2d$
t_1
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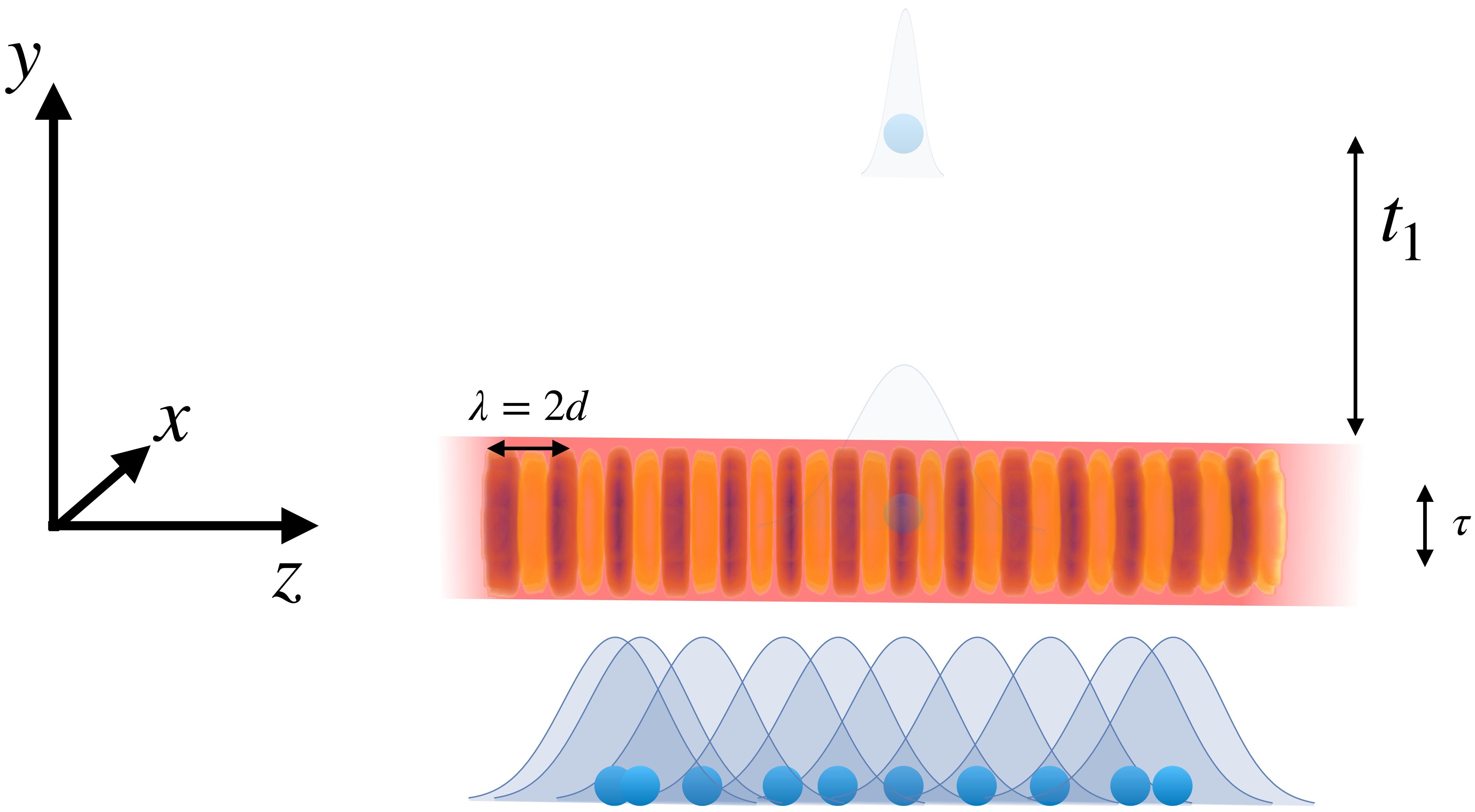

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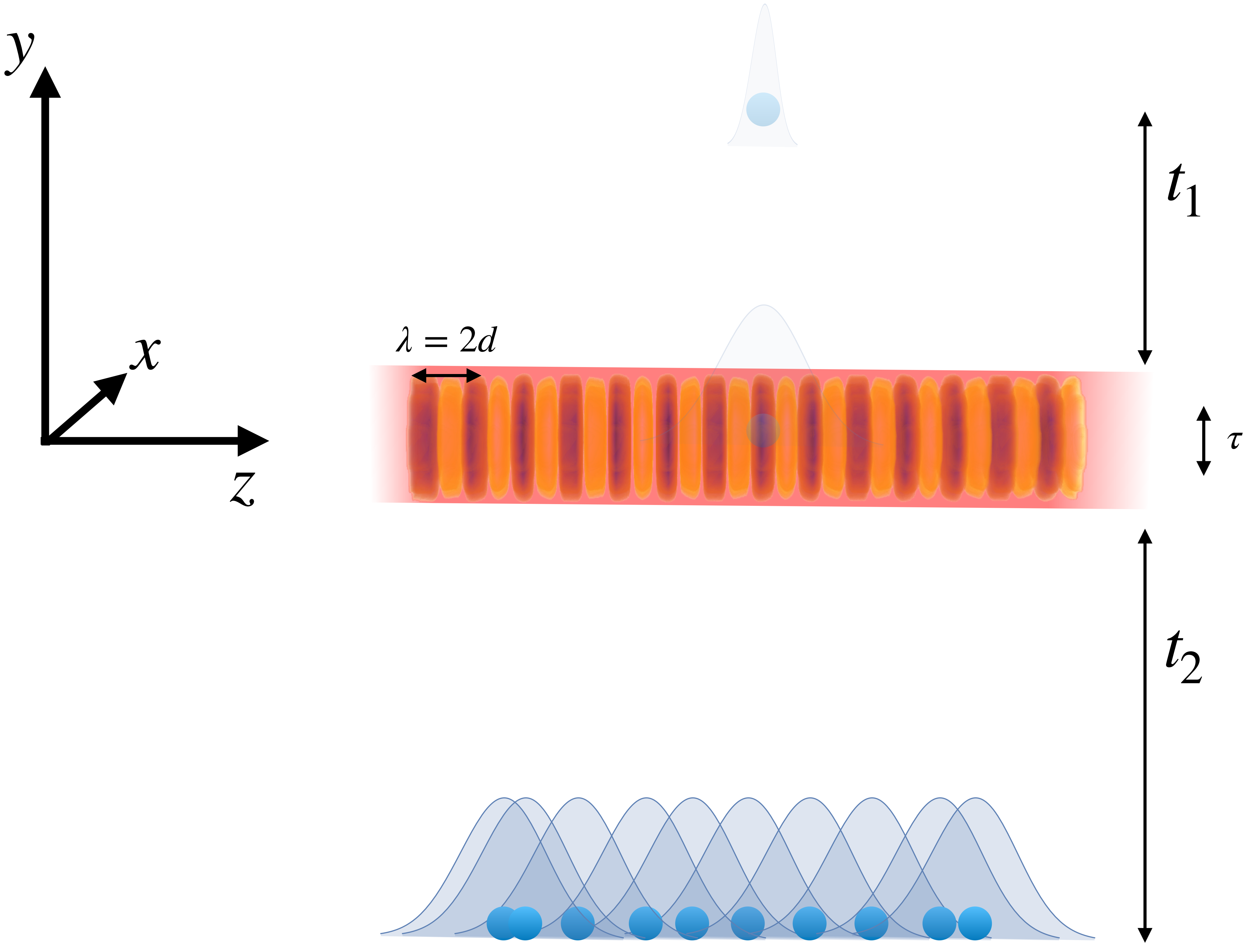
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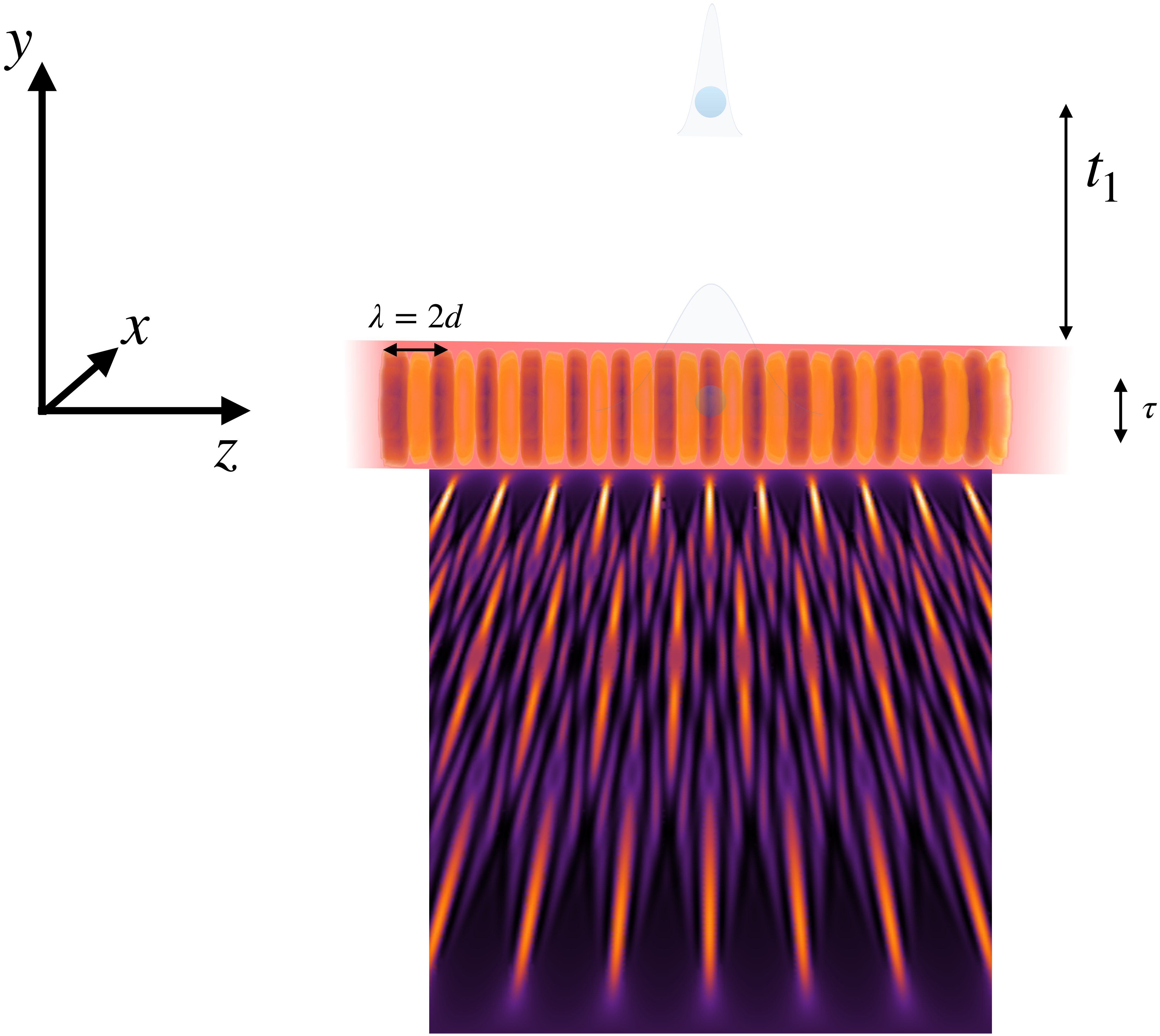

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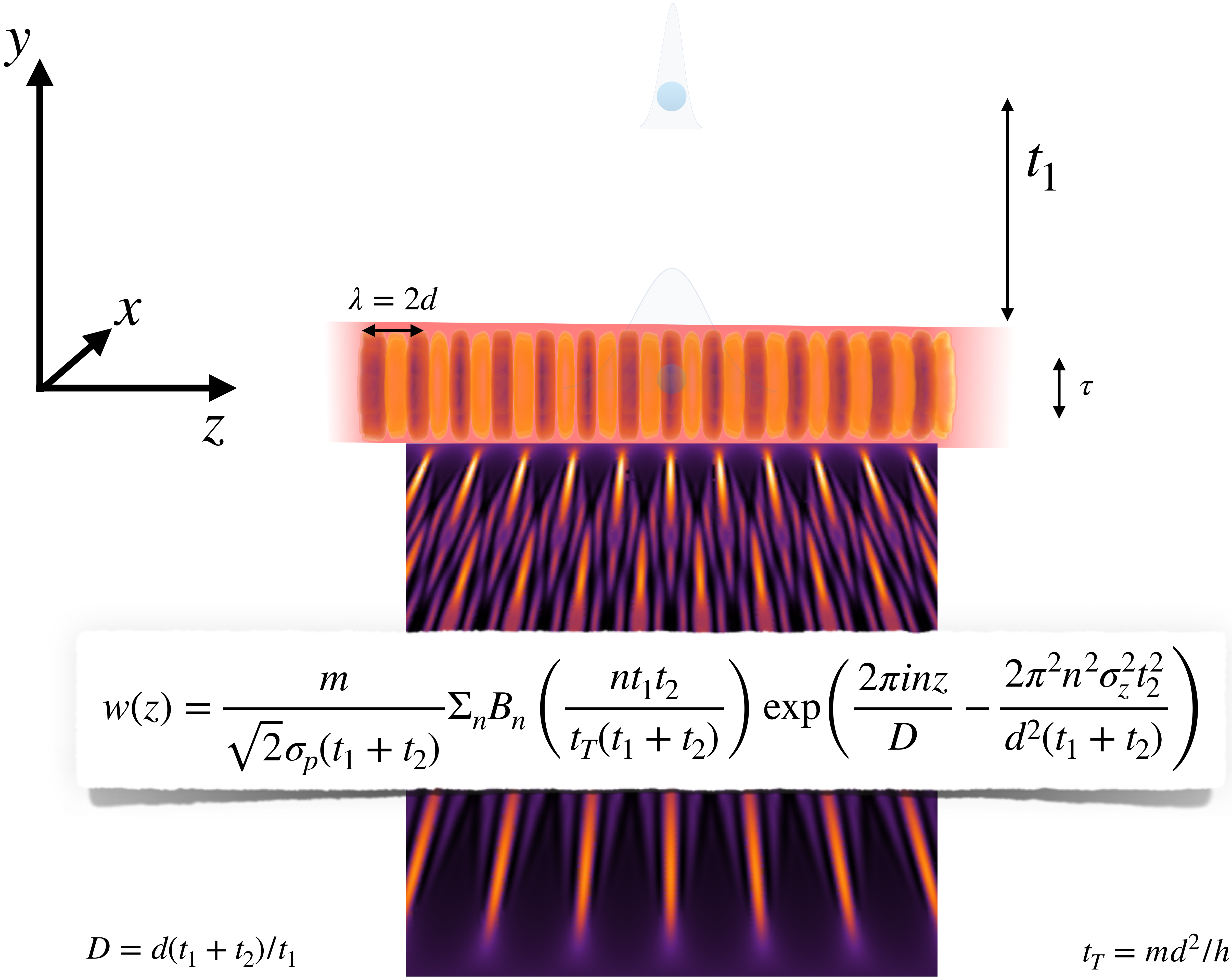

y
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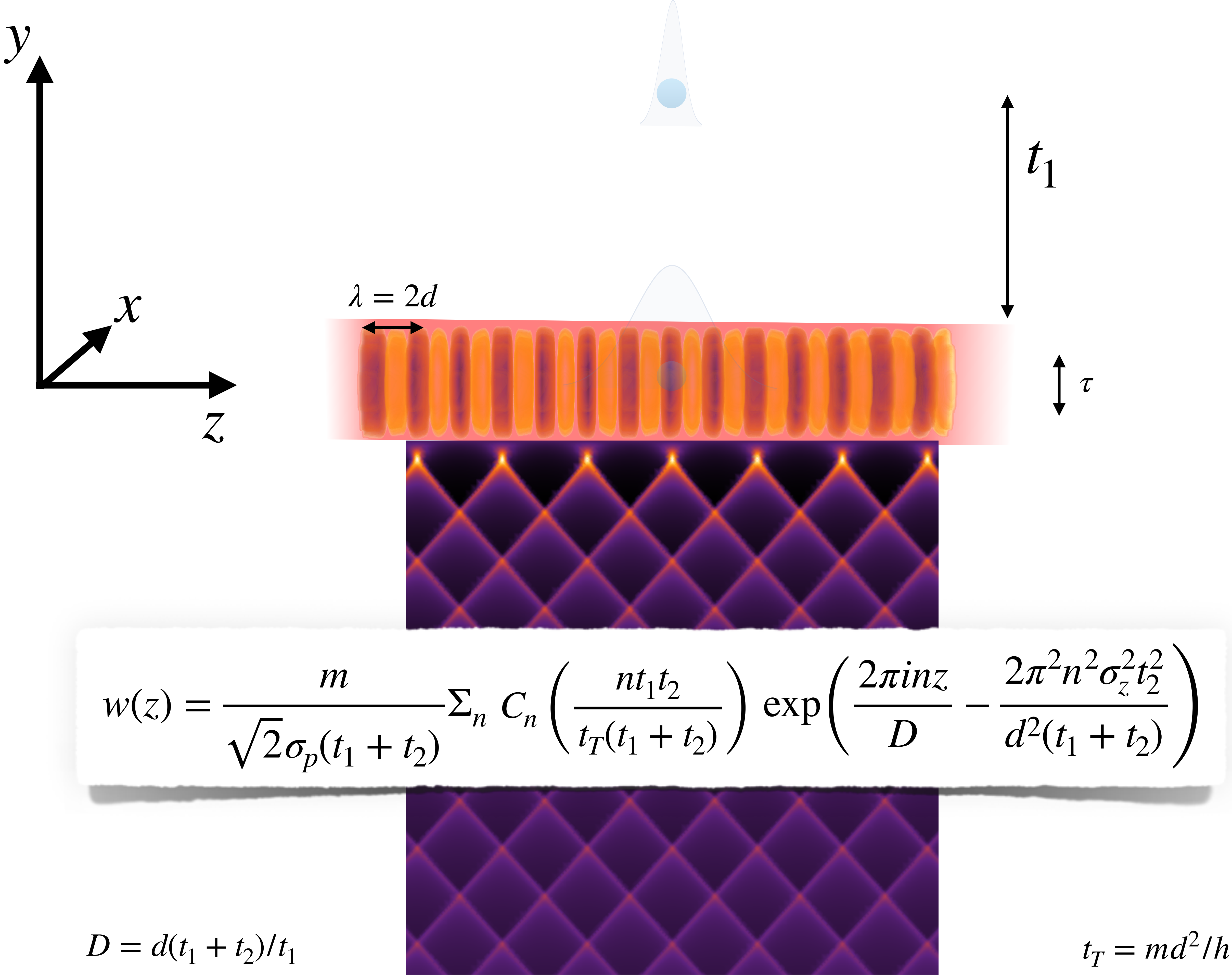
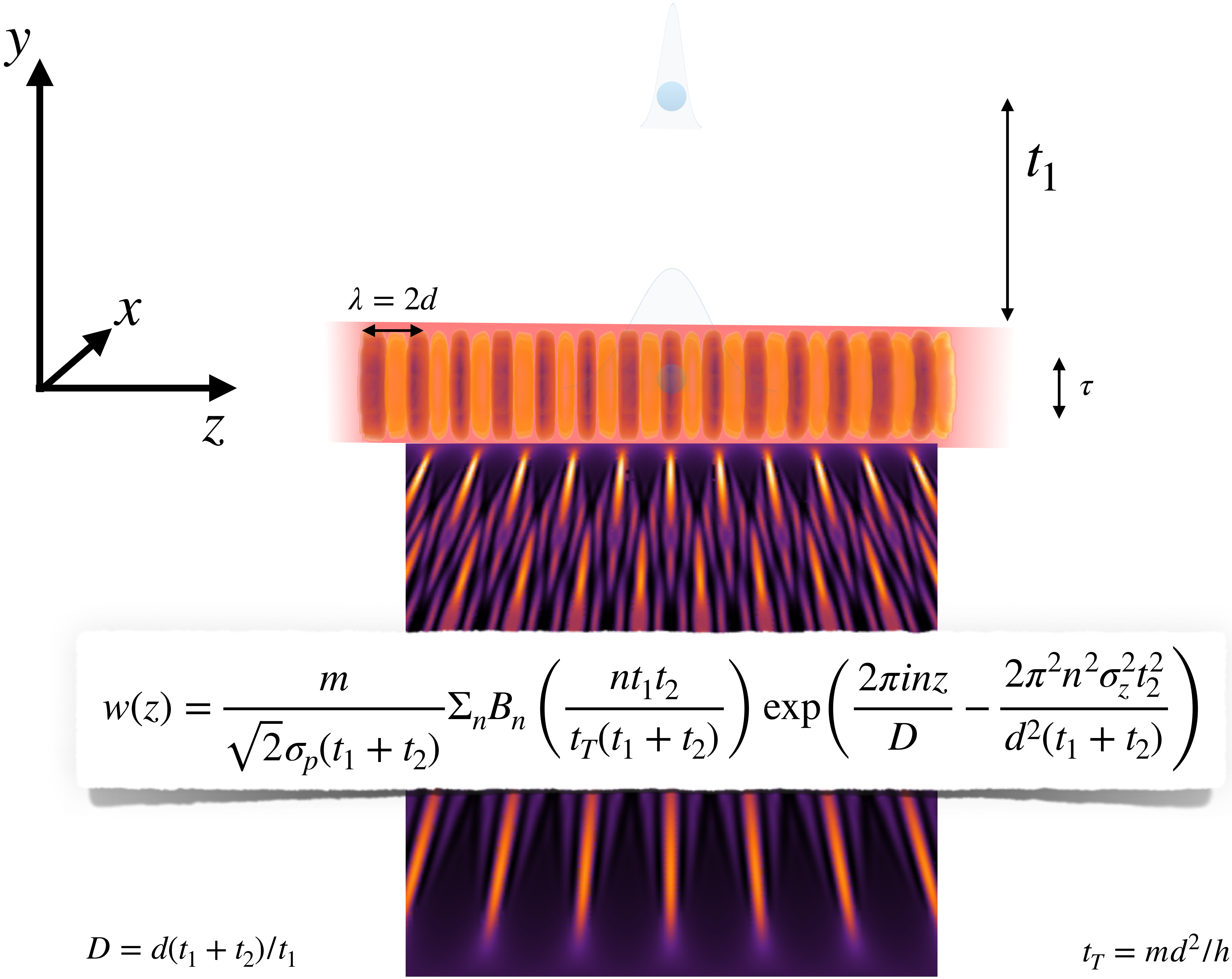
$$w(z) = \frac{m}{\sqrt{2}\sigma_p(t_1+t_2)}\Sigma_n B_n\left(\frac{n t_1 t_2}{t_T(t_1+t_2)}\right)\exp\left(\frac{2\pi i n z}{D} - \frac{2\pi^2 n^2 \sigma_z^2 t_2^2}{d^2(t_1+t_2)}\right)$$
$D = d(t_1+t_2)/t_1$
$t_T = md^2/h$



$$w(z) = \frac{m}{\sqrt{2}\sigma_p(t_1 + t_2)} \Sigma_n \, C_n\left(\frac{n t_1 t_2}{t_T(t_1 + t_2)}\right) \exp\left(\frac{2\pi i n z}{D} - \frac{2\pi^2 n^2 \sigma_z^2 t_2^2}{d^2(t_1 + t_2)}\right)$$



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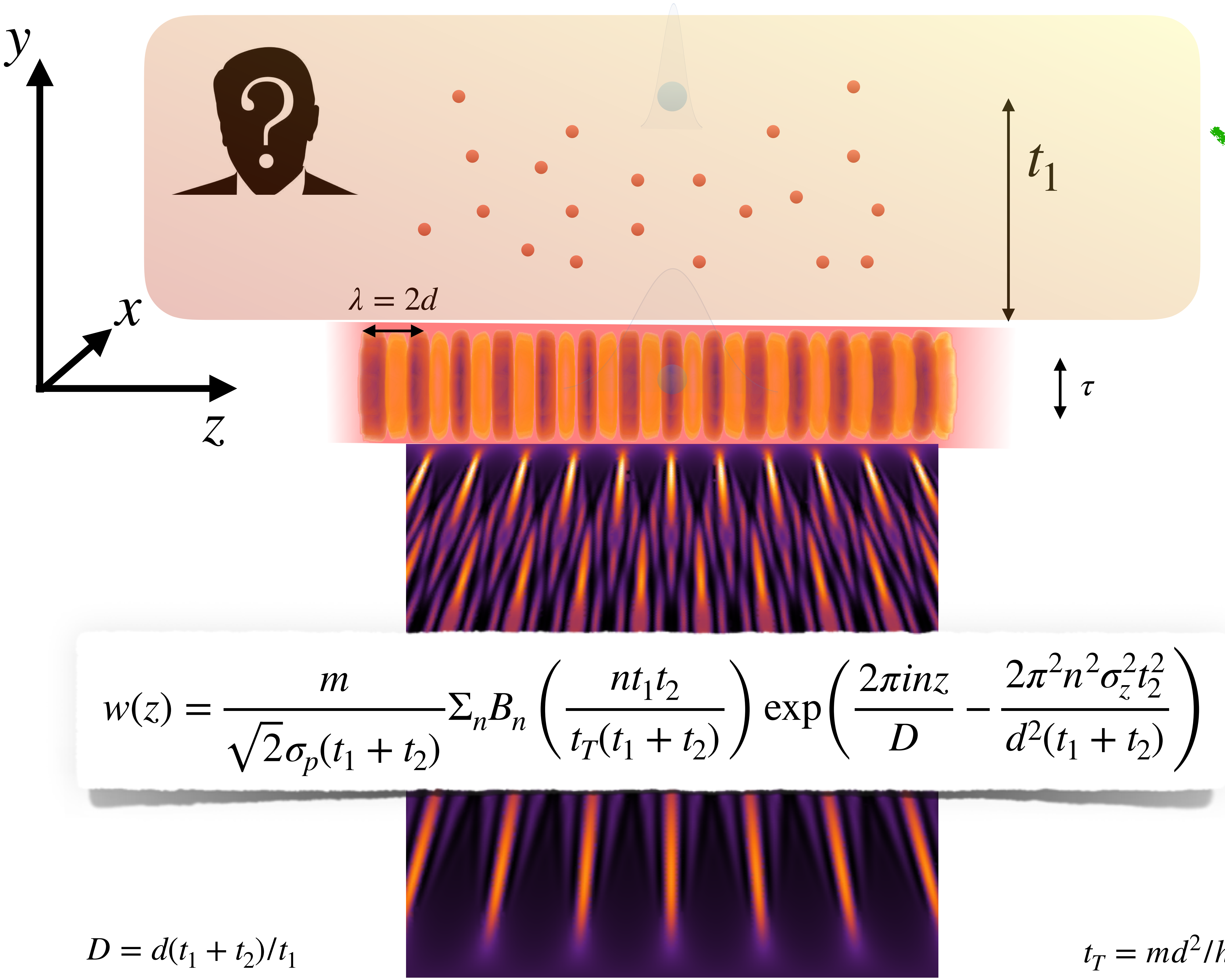
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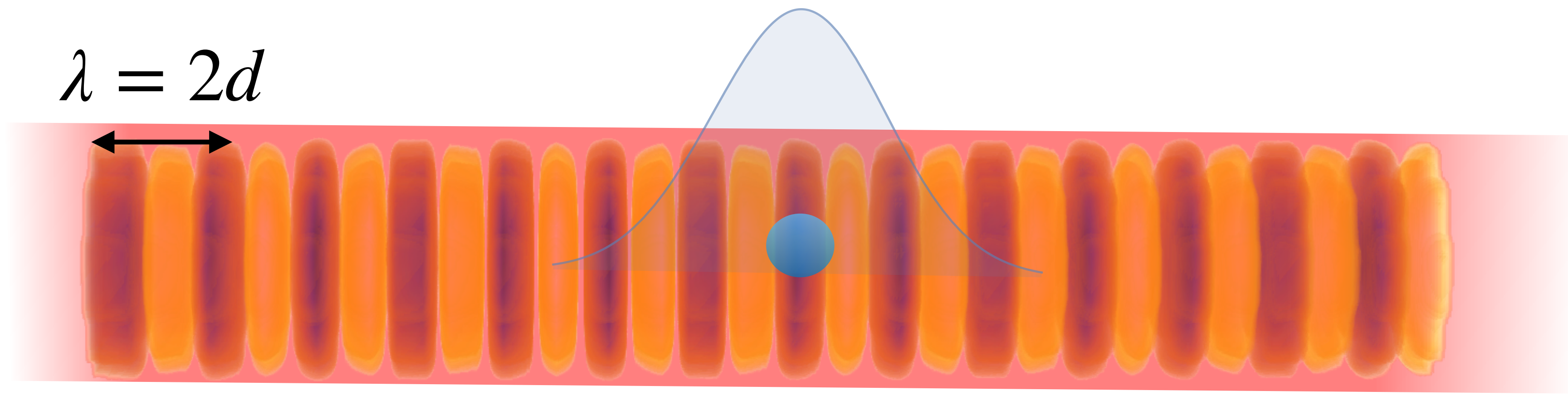


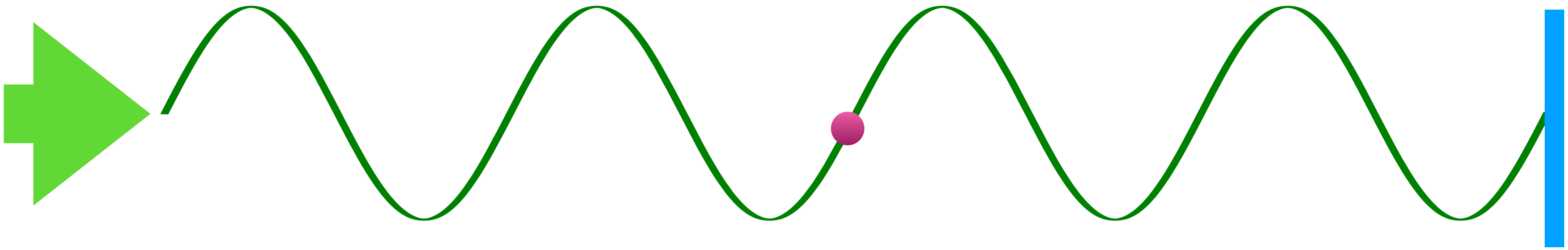
$$w(z) = \frac{m}{\sqrt{2}\sigma_p(t_1+t_2)} \Sigma_n B_n \left(\frac{n t_1 t_2}{t_T(t_1+t_2)}\right) \exp\left(\frac{2\pi i n z}{D} - \frac{2\pi^2 n^2 \sigma_z^2 t_2^2}{d^2(t_1+t_2)}\right)$$

1. Coherent grating for large particles
2. Decoherence effects of Grating Scattering and Absorption

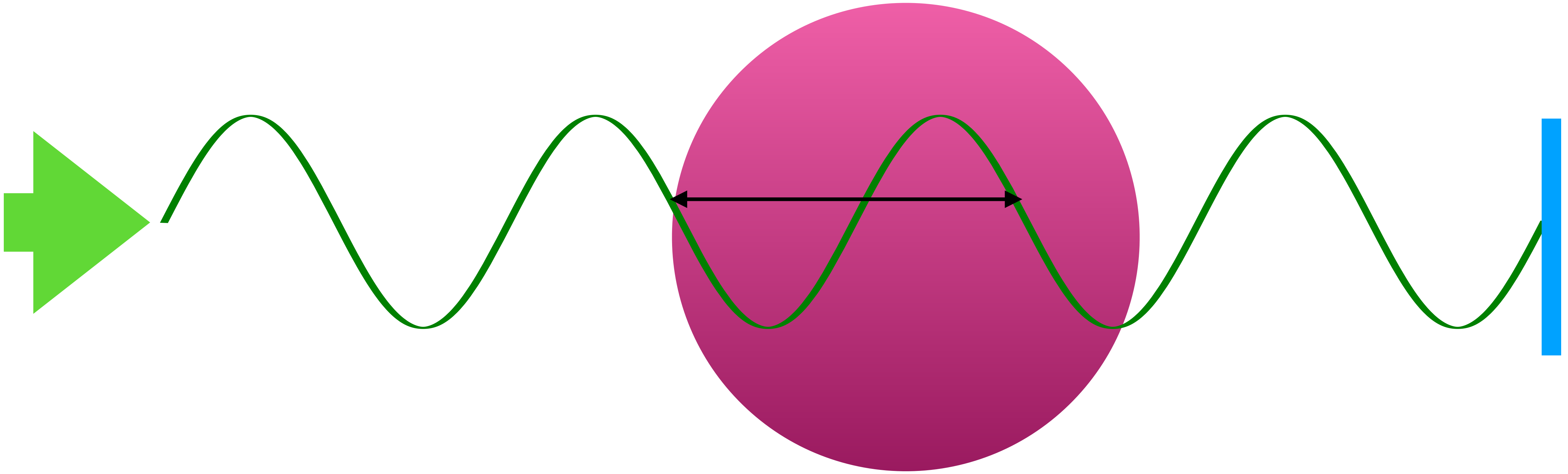
Large (particles) with respect to what?

credit: Skrekkogle art design group

$$kR \ll 1$$



$$kR \geq 1$$



Take at home message:

Increasing the mass of the particles can lead outside the range of validity of Rayleigh approximation and calls for a accurate analysis of grating decoherent effects using Mie scattering theory

General Idea: coherent and incoherent masks

$$\partial_t \rho = -\frac{i}{\hbar}[V, \rho] + \mathscr{L}(\rho)$$

Coherent **In-coherent**

General Idea: coherent and incoherent masks

$$\partial_t \rho = -\frac{i}{\hbar}[V, \rho] + \mathscr{L}(\rho)$$

Coherent **In-coherent**

$$\rho(z, z') \rightarrow R(z, z')T(z, z')\rho(z, z')$$

General Idea: coherent and incoherent masks

$$\partial_t \rho = -\frac{i}{\hbar}[V, \rho] + \mathscr{L}(\rho)$$

Coherent **In-coherent**

$$\rho(z, z') \to R(z, z')T(z, z')\rho(z, z')$$

$$T(z, z') = \exp\left[-\frac{i}{\hbar}\int_0^{\tau_{int}} d\tau (V(z, \tau) - V(z', \tau))\right]$$

$$R(z, z') = \exp \int_0^{\tau_{int}} d\tau \mathscr{L}(z, z')$$

General Idea: phase-space description

$$\rho(z, z') \to R(z, z')T(z, z')\rho(z, z')$$

Wigner function

$$w'(z, p) = \int dq \tilde{T}(z, p - q) w(z, q)$$

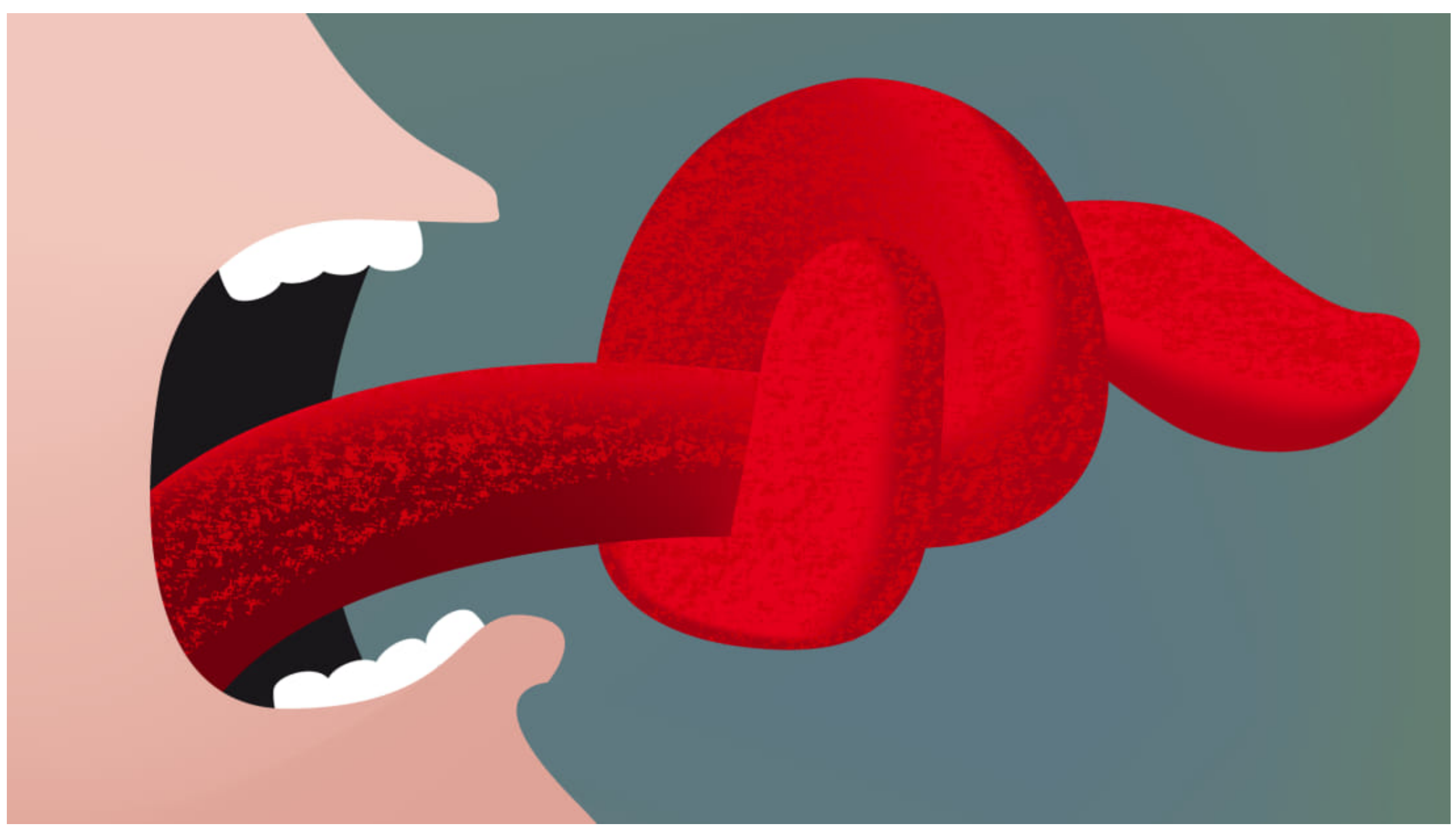
General Idea: phase-space description

$$\rho(z,z') \rightarrow R(z,z')T(z,z')\rho(z,z')$$

Wigner function

$$w'(z,p) = \int dq \tilde{T}(z,p-q)w(z,q)$$

Convolution with a convolution kernel which is the convolution of two kernels



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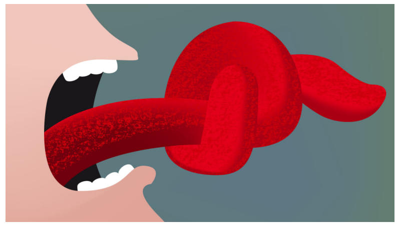
Convolution with a convolution kernel which is the convolution of two kernels



$$\tilde{T}(z,p) = \int dq \mathscr{R}(z,p-q)\mathscr{T}_{\text{coh}}(z,q)$$

General Idea: phase-space description

Convolution with a convolution kernel which is the convolution of two kernels

$$\tilde{T}(z,p) = \int dq \mathscr{R}(z, p-q)\mathscr{T}_{\text{coh}}(z,q)$$

$$\mathscr{R} = \frac{1}{2\pi\hbar}\int ds e^{ips/\hbar} R(z - s/2, z + s/2)$$

$$\mathscr{T}_{\text{coh}} = \frac{1}{2\pi\hbar}\int ds e^{ips/\hbar} T(z - s/2, z + s/2)$$

General Idea: phase-space description

Convolution with a convolution kernel which is the convolution of two kernels

$$\tilde{T}(z,p) = \int dq \mathscr{R}(z,p-q)\mathscr{T}_{\text{coh}}(z,q)$$

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$$\mathscr{T}_{\text{coh}} = \frac{1}{2\pi\hbar}\int ds e^{ips/\hbar} T(z-s/2, z+s/2)$$

$$\mathscr{T}_{\text{coh}} = \frac{1}{2\pi\hbar}\sum_n e^{2\pi i n z/d}\int ds e^{iqs/\hbar} B_n(s/d)$$

General Idea: phase-space description

Convolution with a convolution kernel which is the convolution of two kernels

$$\tilde{T}(z,p) = \int dq \mathscr{R}(z, p-q)\mathscr{T}_{\mathrm{coh}}(z,q)$$

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$$\tilde{T}(z,p) = \frac{1}{2\pi\hbar}\sum_n e^{2\pi i n z/d}\int ds e^{ips/\hbar}\tilde{B}_n(s/d)$$

General Idea: phase-space description

$$\tilde{T}(z,p) = \frac{1}{2\pi\hbar} \sum_n e^{2\pi i n z/d} \int ds e^{ips/\hbar} \tilde{B}_n(s/d)$$

$$\tilde{B}_n(\xi) = \sum_j B_{n-j}(\xi) R_j(\xi)$$

$$R_n(\xi) = \frac{1}{d} \int_{-d/2}^{d/2} dx R(x - \xi d/2, x + \xi d/2) \exp(-2\pi i n x/d)$$

General Idea: phase-space description

$$\tilde{B}_n(\xi) = \sum_j B_{n-j}(\xi) R_j(\xi)$$

$$w(z) = \frac{m}{\sqrt{2}\sigma_p(t_1 + t_2)} \Sigma_n \tilde{B}_n \left(\frac{n t_1 t_2}{t_T(t_1 + t_2)} \right) \exp\left(\frac{2\pi i n z}{D} - \frac{2\pi^2 n^2 \sigma_z^2 t_2^2}{d^2(t_1 + t_2)} \right)$$

Coherent grating for ~~large~~ particles

$$kR \ll 1$$

Dipole Potential

$$V(z,t) = -\frac{1}{4} Re(\chi) |\mathbf{E}(z,t)|^2$$

$$\chi = 4\pi\epsilon_0 R^3 \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 2} = \epsilon_0 \epsilon_c V$$

Standing wave

Use (longitudinal) eikonal approximation

$$\langle z | \psi \rangle \rightarrow \exp(i\phi_0 \cos^2 kz) \langle z | \psi \rangle$$

$$\phi(z) = \frac{1}{\hbar} \int_\tau dt V(z,t) = \phi_0 \cos^2 kz$$

Coherent grating for ~~large~~ particles

$kR \ll 1$

Use (longitudinal) eikonal approximation

$$\langle z | \psi \rangle \rightarrow \exp(i\phi_0 \cos^2 kz)\langle z | \psi \rangle$$

$$\phi(z) = \frac{1}{\hbar}\int_\tau dt V(z,t) = \phi_0 \cos^2 kz$$

$$\phi_0 = \frac{2Re(\chi)E_L}{\hbar c \epsilon_0 a_L}$$

S. Nimmrichter, *Macroscopic matter wave interferometry*. Springer, 2014

Coherent grating for large particles $kR \sim 1$

The light-induced forces acting on the dielectric particle can be obtained by integrating the electromagnetic stress-energy tensor over a spherical surface surrounding the particle.

$$\frac{F_z(z)}{I_0 k^{-2} c^{-1}} = -(kR)^4 \sum_{\ell=1}^{\infty} \sum_{m=\pm 1} \mathrm{Im}\left[\ell(\ell+2)\sqrt{\frac{(\ell-\mathrm{m}+1)(\ell+\mathrm{m}+1)}{(2\ell+3)(2\ell+1)}} \times (2a_{\ell+1,m}a^*_{\ell m} + a_{\ell+1,m}A^*_{\ell m} + A_{\ell+1,m}a^*_{\ell m} + 2b_{\ell+1,m}b^*_{\ell m} + b_{\ell+1,m}B^*_{\ell m} + B_{\ell+1,m}b^*_{\ell m}) + m(2a_{\ell,m}b^*_{\ell m} + a_{\ell,m}B^*_{\ell m} + A_{\ell,m}b^*_{\ell m})\right],$$

Longitudinal force on a dielectric sphere in vacuum

$$F_z(z) = -F_0 \sin 2kz$$

$$V(z) = -(F_0/2k)\cos 2kz$$

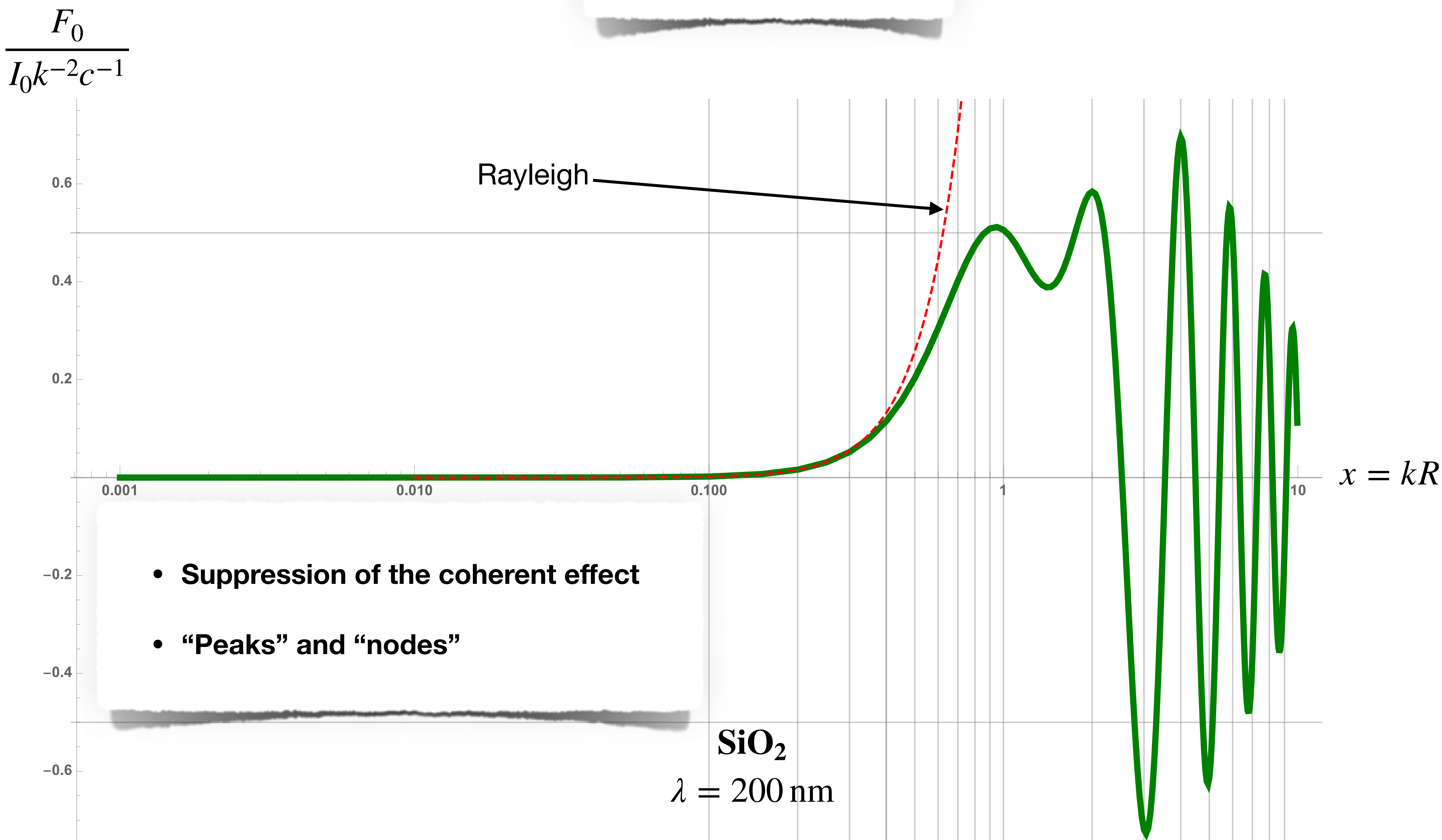
$$\phi_0 = \frac{8F_0E_L}{\hbar c \epsilon_0 a_L k |E_0|^2}$$

S. Nimmrichter, *Macroscopic matter wave interferometry*. Springer, 2014

J. Barton, D. Alexander and S. Schaub, *Theoretical determination of net radiation force and torque for a spherical particle illuminated by a focused laser beam*, Journal of Applied Physics 66 (1989)

Coherent grating for large particles $kR \sim 1$

$$\phi_0 = \frac{8F_0E_L}{\hbar c\epsilon_0 a_L k|E_0|^2}$$

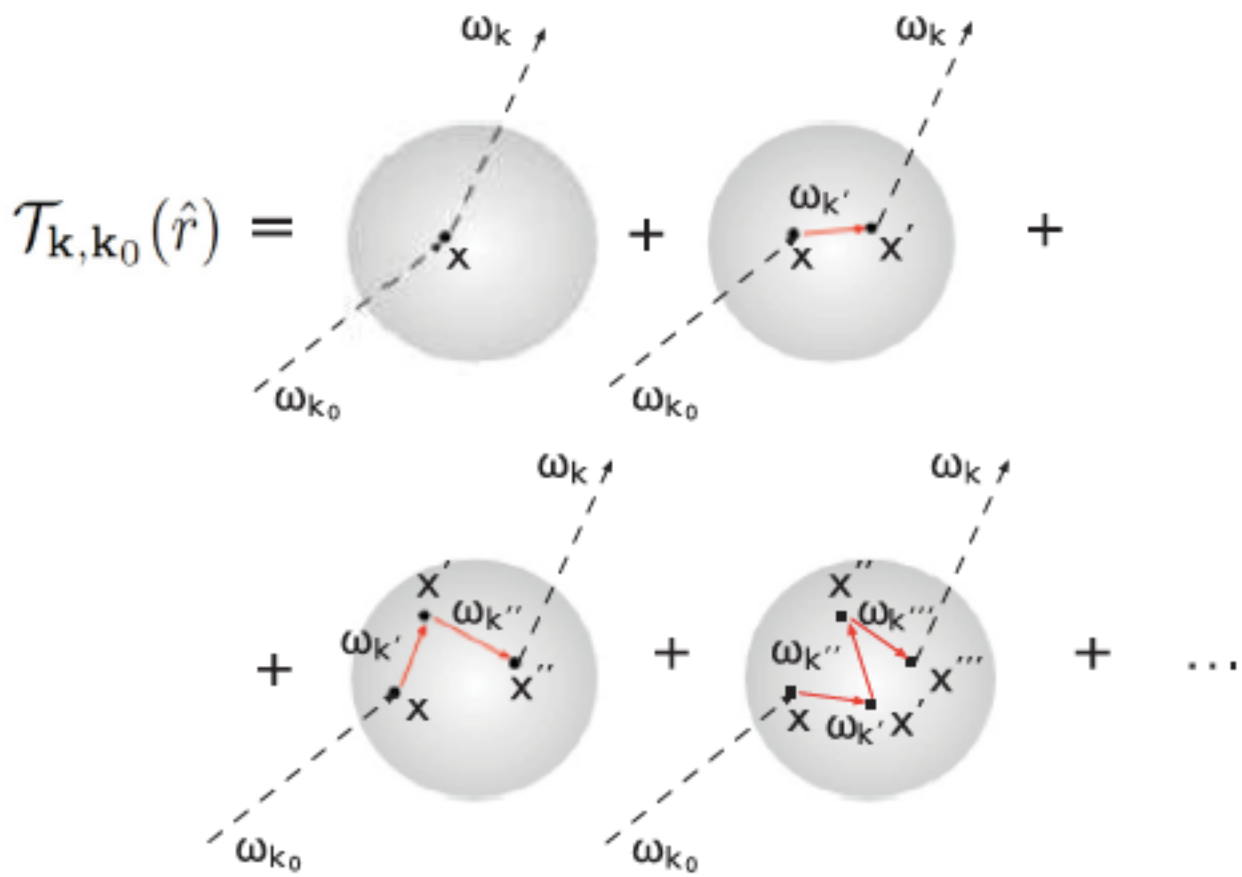


- **Suppression of the coherent effect**
- **“Peaks” and “nodes”**

Incoherent Effects: Scattering

$$\mathscr{L}[\rho_S] = |\alpha|^2 \int d\mathbf{k}\,\delta(\omega_k - \omega_0)\left(2\mathscr{T}_{\mathbf{k}c}(\hat{r})\rho_S \mathscr{T}^*_{c\mathbf{k}}(\hat{r}) - \left\{ |\mathscr{T}_{\mathbf{k}c}(\hat{r})|^2, \rho_S \right\} \right)$$

$$\mathscr{T}_{\mathbf{k},c}(\hat{\mathbf{r}}) = \int d\mathbf{k}' \langle c | \mathbf{k}' \rangle \mathscr{T}^*_{\mathbf{k}',\mathbf{k}}(\hat{\mathbf{r}})$$



A.C. Pflanzer, O. Romero-Isart, and J. I. Cirac, Phys. Rev. A 86, 013802 (2012)

Incoherent Effects: Scattering

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$$\langle z|\rho|z'\rangle \to R(z,z')\langle z|\rho|z'\rangle$$

$$\langle z|e^{\mathscr{L}t}\rho|z'\rangle = \exp\left\{ -\int dt\, |\alpha|^2 \int d\mathbf{k}\delta(\omega_k - \omega_0)\left[-2\mathscr{T}_{\mathbf{k}c}(z)\mathscr{T}^*_{c\mathbf{k}}(z') + |\mathscr{T}_{\mathbf{k}c}(z)|^2 + |\mathscr{T}_{\mathbf{k}c}(z')|^2\right] \right\}\langle z|\rho|z'\rangle$$

A.C. Pflanzer, O. Romero-Isart, and J. I. Cirac, Phys. Rev. A 86, 013802 (2012)

Incoherent Effects: Absorption

$$\mathscr{L}(\rho) = \frac{c\sigma_{\text{abs}}}{V_0} |\alpha(t)|^2 \left[\cos(kz)\rho\cos(kz) - \frac{1}{2}\{\cos^2(kz), \rho\} \right]$$

S. Nimmrichter, *Macroscopic matter wave interferometry*. Springer, 2014

Incoherent Effects: Absorption

$$\mathbf{E}(\mathbf{r}) = f(\mathbf{r})\,\hat{e}_x$$

Standing waves in terms of plane ones

Momentum kick

$$f(\mathbf{r}) = \sum f_{\mathbf{k}} \exp(i\mathbf{k}\cdot\mathbf{r})$$

$$|\,\mathbf{p}\rangle \rightarrow \sum f_{\mathbf{k}}\,|\,\mathbf{p} + \hbar\mathbf{k}\rangle = f(\mathbf{r})\,|\,\mathbf{p}\rangle$$

Jump operator: $f(\mathbf{r}) \sim \cos(\mathbf{kr})$

$$\mathscr{L}(\rho) = \frac{c\sigma_{\mathrm{abs}}}{V_0}\,|\,\alpha(t)\,|^2\left[\cos(kz)\rho\cos(kz) - \frac{1}{2}\{\cos^2(kz),\rho\}\right]$$

$$\sigma_{\mathrm{abs}} = \sigma_{\mathrm{ext}} - \sigma_{\mathrm{sca}} = \frac{2\pi}{k^2}(2n+1)\Sigma_{n=1}^{\infty}\left(\mathrm{Re}\left(a_n + b_n\right) - |\,a_n\,|^2 - \left|b_n\right|^2\right)$$

S. Nimmrichter, *Macroscopic matter wave interferometry*. Springer, 2014

Generalized Talbot Coefficients

$$\tilde{B}_n(\xi) = \exp(F - c_{\text{abs}}/2) \sum_{k=-\infty}^{\infty} \left(\frac{\zeta_{\text{coh}}(\xi) + a + c_{\text{abs}}/2}{\zeta_{\text{coh}}(\xi) - a - c_{\text{abs}}/2} \right)^{\frac{n+k}{2}} J_{n+k}\left(\text{sign}(\zeta_{\text{coh}} - \text{a} - \text{c}_{\text{abs}}/2)\sqrt{\zeta_{\text{coh}}^2 - (\text{a} + \text{c}_{\text{abs}}/2)^2} \right) J_k(b)$$

Absorption

$$c_{\text{abs}} = n_0(1 - \cos(\pi\xi)) \qquad n_0 = \frac{4\sigma_{\text{abs}}}{hc}\frac{E_L}{a_L}\lambda = \frac{I_0}{ck^2F_0}\sigma_{\text{abs}}k^2\phi_0$$

$$\zeta_{coh} = \phi_0 \sin(\pi\xi)$$

Scattering

$$a = 2\pi^2 \int dt \frac{|\alpha|^2 c}{4\pi^2 V_0} \int d\Omega\, Re\left(f^*(\mathbf{k}_0, k_0\mathbf{n}) f(-\mathbf{k}_0, k_0\mathbf{n})\right) \left[(\cos(\pi n_z \xi) - \cos(\pi\xi)) \right]$$

$$b = i\, 2\pi^2 \int dt \frac{|\alpha|^2 c}{4\pi^2 V_0} \int d\Omega\, Im\left(f^*(\mathbf{k}_0, k_0\mathbf{n}) f(-\mathbf{k}_0, k_0\mathbf{n})\right) \left[\sin(\pi n_z \xi) \right]$$

$$F = 2\pi^2 \int dt \frac{|\alpha|^2 c}{4\pi^2 V_0} \int d\Omega\, |f(\mathbf{k}_0, k_0\mathbf{n})|^2 \left[\left(\cos(\pi n_z\xi)\cos(\pi\xi) + \sin(\pi n_z\xi)\sin(\pi\xi)\right) - 1 \right]$$

Let's take a Look

Laser: $\lambda = 2d = 354 \times 10^{-9}$m		
Material: Si		
$\rho_{\mathrm{Si}} = 2.3290 \times 10^3 \mathrm{Kg/m^3}$	$T = 20 \times 10^{-3}$K	
Refractive Index at λ: $n = 5.656 + i\,2.952$		
Trapping frequency: $\nu = 200 \times 10^3$Hz		
Interferometer:		
$d = 177 \times 10^{-9}$m	$t_1 = 2t_T$	$t_2 = 1.6t_T$

Table 1. Parameters considered for Si spheres.

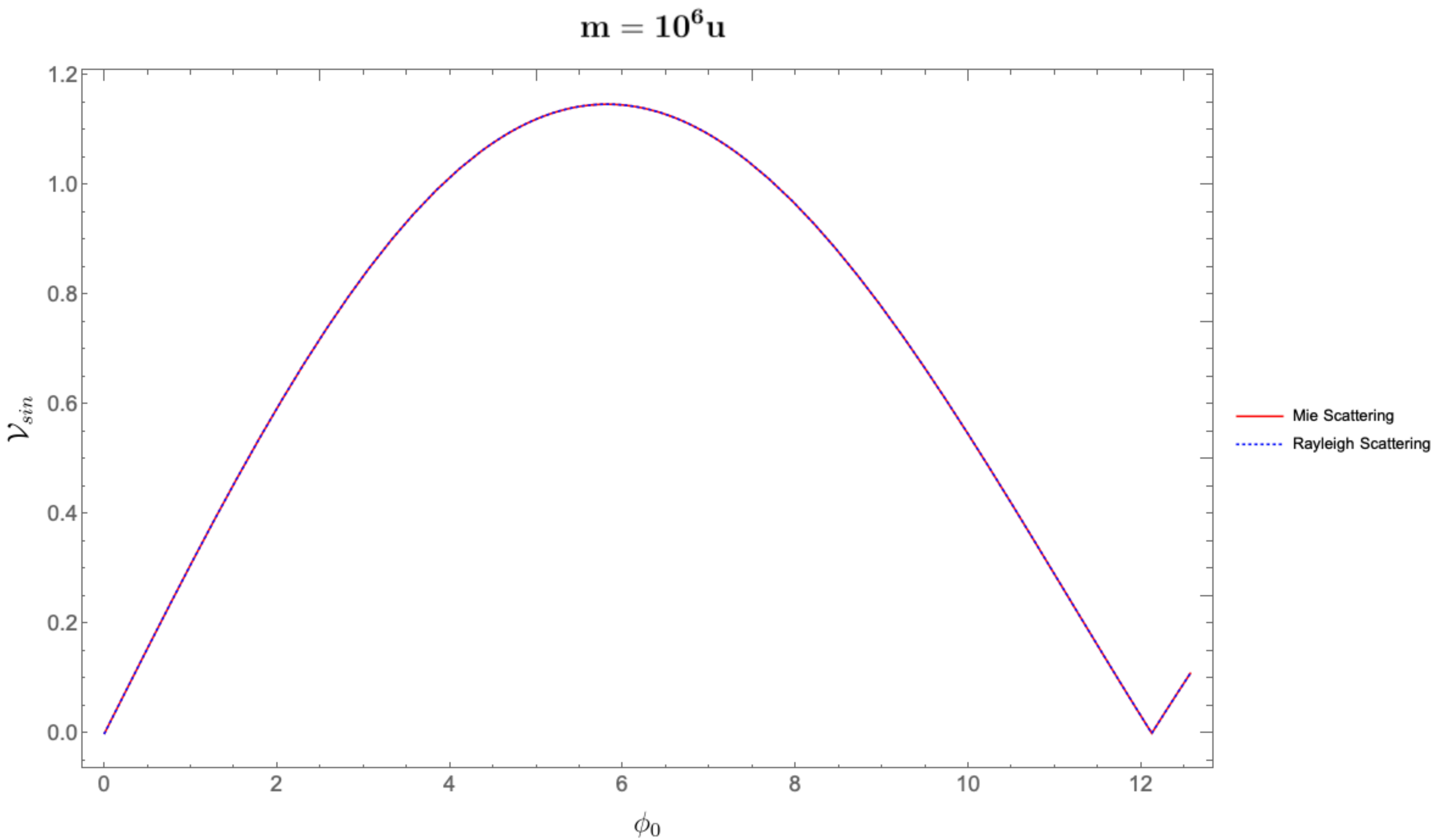
$$R = \left(\frac{3}{4\pi}\frac{m}{\rho_{Si}}\right)^{1/3}$$

$m = 10^6\,\mathrm{u}$	$R \sim 5.54\,\mathrm{nm}$	$kR \sim 0.098$
$m = 10^8\,\mathrm{u}$	$R \sim 25.71\,\mathrm{nm}$	$kR \sim 0.46$

J. Bateman, S. Nimmrichter, K. Hornberger and H. Ulbricht, Nature communications 5 (2014) 4788

Let's take a Look

$m = 10^6\,\mathrm{u}$	$R \sim 5.54\,\mathrm{nm}$	$kR \sim 0.098$



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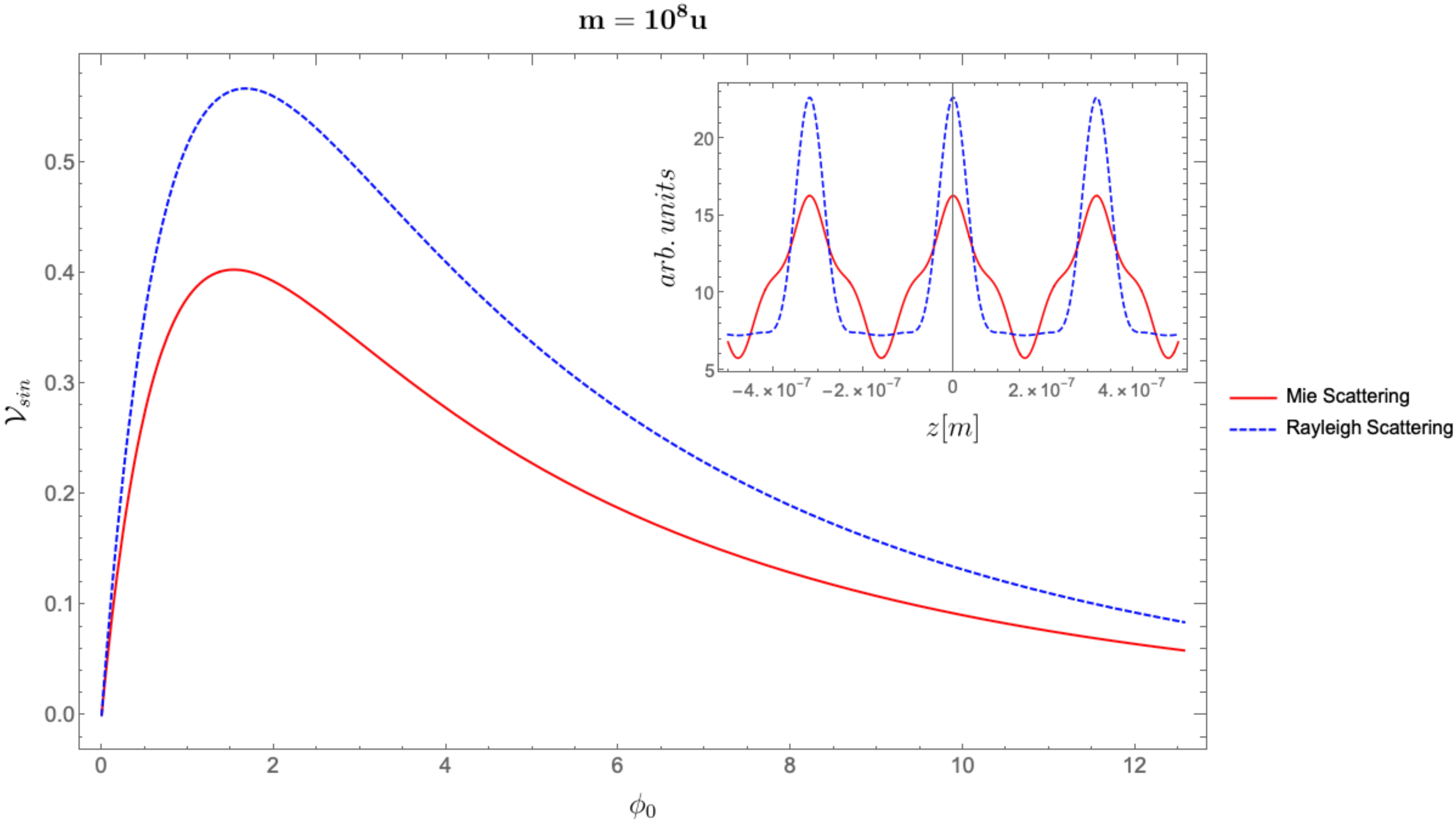

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Conclusions

- **Coherent grating is strongly affected by the size of the particles**

- **Decoherent effects are also strongly affected by the size**

- **Need to take both aspects into account for the best theoretical modelling of the interference pattern**

- **Classical limit of the in-coherent effects is not trivial and deserves an in-depth investigation**


thank you
danke
謝謝
рахмат
ngiyabonga
teşekkür ederim
gracias
merci
obrigado
bedankt
dziękuję
спасибо
Баярлалаа
go raibh maith agat
mochchakkeram
sukriya
kop khun krap
terima kasih
감사합니다
ありがとう
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vinaka
спасибі
blagodaram
misaotra
matondo
paldies
grazzi
mahalo
хвала
asante
manana
obrigada
tenki
murakoze
chokrane
mamnun
дякую
djiere dieuf
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kiitos
dankie
nanni
nandri
dhanyavad
hvala
mauruuru
köszönöm
gràçie
bayarlalaa
enkosi
akun
dankon
aćiū
chnorakaloutioun
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sulpáy
taiku
sobodi
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najis tuke
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