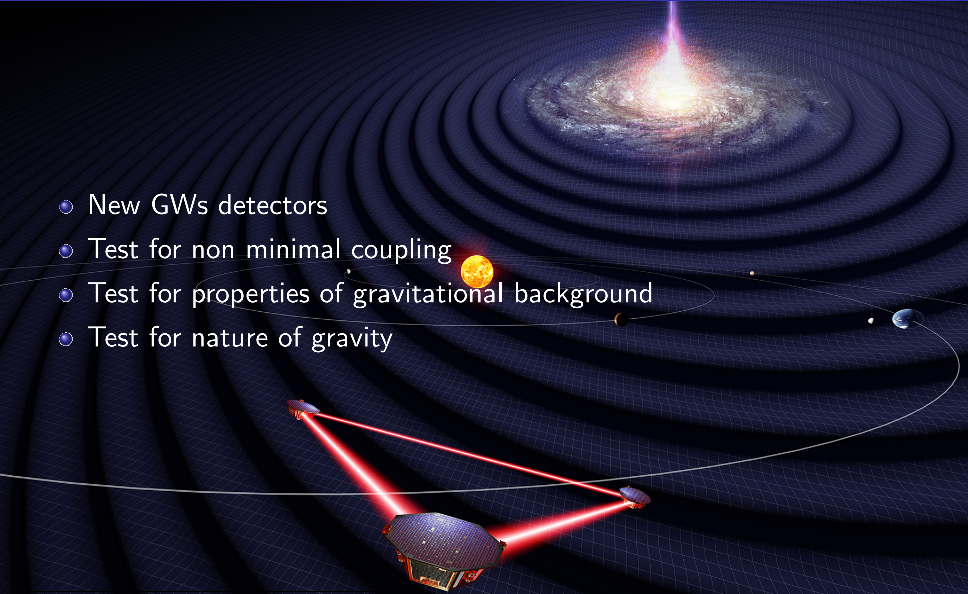
Gravitational decoherence: a general non relativistic model

Lorenzo Asprea

Trieste junior quantum days - ICTP, Adriatico building

26th July 2019

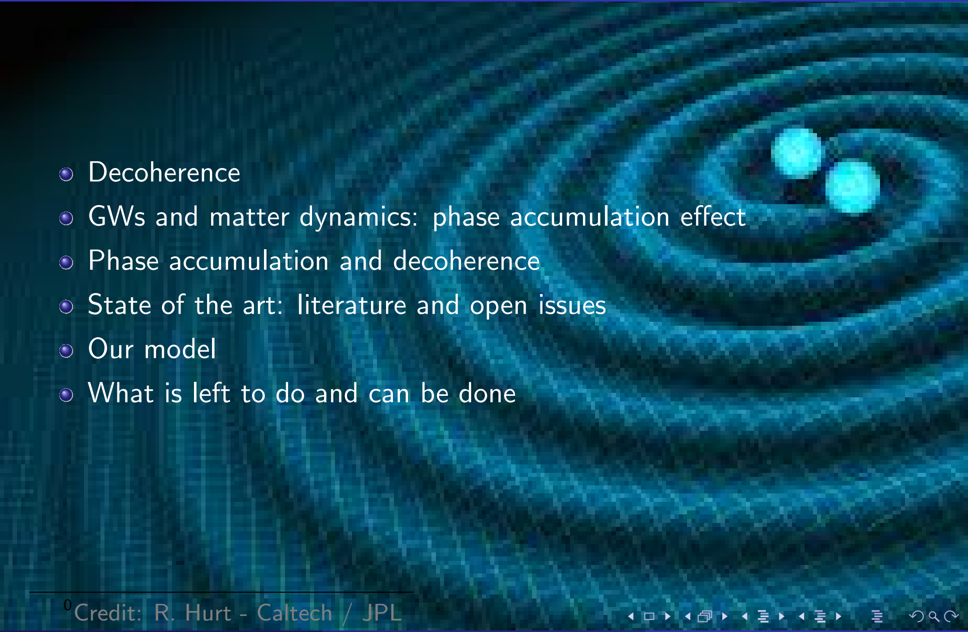
Motivation



- New GWs detectors
- Test for non minimal coupling
- Test for properties of gravitational background
- Test for nature of gravity

[0] Credit: University of Florida / S. Barke

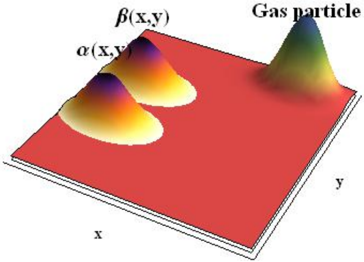
Outlook



- Decoherence
- GWs and matter dynamics: phase accumulation effect
- Phase accumulation and decoherence
- State of the art: literature and open issues
- Our model
- What is left to do and can be done

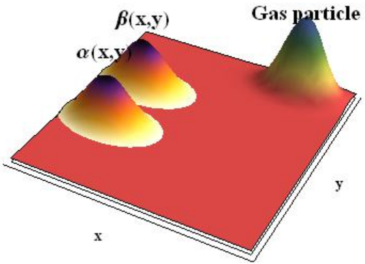
[0] Credit: R. Hurt - Caltech / JPL

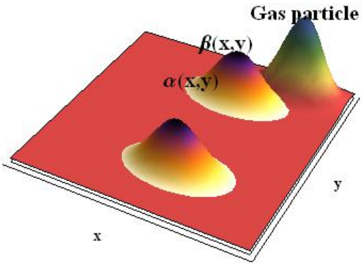
Decoherence



$$\psi(x, y, 0) = \frac{(\alpha(x,y,0)+\beta(x,y,0))}{\sqrt{2}} \otimes \chi(x, y, 0)$$

Decoherence

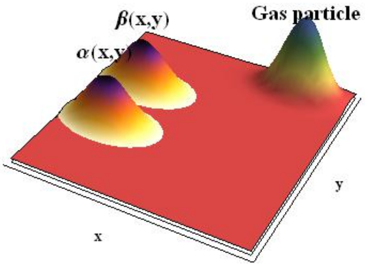


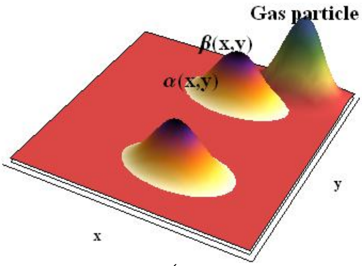


$$\psi(x,y,0) = \frac{(\alpha(x,y,0)+\beta(x,y,0))}{\sqrt{2}} \otimes \chi(x,y,0)$$

$$\psi(x,y,t) = \frac{1}{\sqrt{2}}\Big(\alpha(x,y,t)\otimes\chi_\alpha(x,y,t) + \\ +\beta(x,y,t)\otimes\chi_\beta(x,y,t)\Big)$$

Decoherence



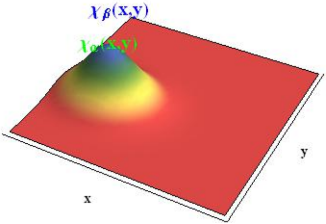


$$\psi(x,y,0)=\frac{(\alpha(x,y,0)+\beta(x,y,0))}{\sqrt{2}}\otimes\chi(x,y,0)\quad \psi(x,y,t)=\frac{1}{\sqrt{2}}\Big(\alpha(x,y,t)\otimes\chi_\alpha(x,y,t)+ +\beta(x,y,t)\otimes\chi_\beta(x,y,t)\Big)$$

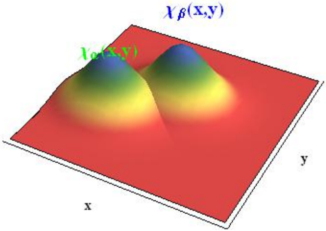
$$\begin{aligned}\rho_s(x,y,t)&=\langle x|\,Tr^E[|\psi(t)\rangle\langle\psi(t)|]|y\rangle\\&=\langle\chi_\alpha(t)|\rho_\psi(x,y,t)|\chi_\alpha(t)\rangle+\langle\chi_\beta(t)|\rho_\psi(x,y,t)|\chi_\beta(t)\rangle\end{aligned}$$

Decoherence

$$\rho_s(x,y,t) = \langle x|\, Tr^E[|\psi(t)\rangle\langle\psi(t)|]\,|y\rangle$$
$$= \frac{1}{2}\begin{pmatrix} 1 & \alpha^*(x,y,t)\beta(x,y,t)\langle\chi_\alpha(t)|\chi_\beta(t)\rangle \\ \beta^*(x,y,t)\alpha(x,y,t)\langle\chi_\beta(t)|\chi_\alpha(t)\rangle & 1 \end{pmatrix}$$



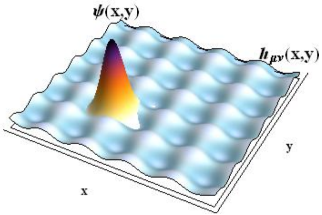
t = 0



t > 0

$$\rho_s(x,y,t \gg 1) \sim \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Gravitational waves and matter dynamics

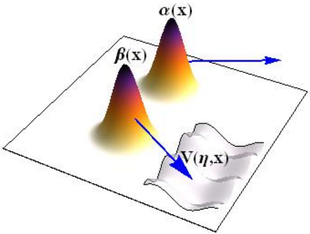


- KG equation: $g^{\mu\nu}\nabla_\mu\partial_\nu\psi - \frac{m^2c^2}{\hbar^2}\psi = 0$
- WKB approximation:$\psi(x) = e^{i\phi(x)/\hbar}$
  $\Rightarrow g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \frac{m^2c^2}{\hbar^2}\phi$
- weak field $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $h_{\mu\nu} \ll 1$
- long λ_{gw} (pointlike wavefunction)
- expand $\phi(x)$ in powers of $h_{\mu\nu}$:
  $\phi(x) = \phi_{(0)}(x) + \phi_{(1)}(x)$

Phase accumulation equation

$$\frac{d\phi_{(1)}}{d\tau} = \frac{1}{2\hbar^2}h_{\mu\nu}p^\mu p^\nu$$

Phase accumulation and decoherence



- $\psi(\overline{x},0) = \frac{(\alpha(\overline{x},0)+\beta(\overline{x},0))}{\sqrt{2}}$
- $H_{int} = V(\overline{x},\eta)$

Aharonov Bohm effect: phase accumulation in external potential

$$\varphi(\eta) = -\int dt V(\overline{x}(t),\eta)$$

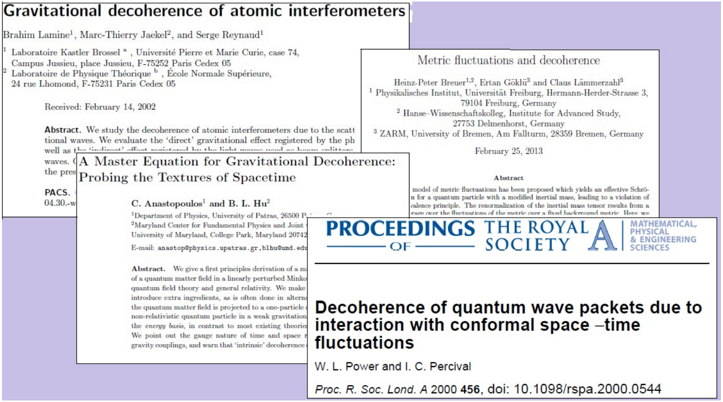
- $\psi(\overline{x},t) = \frac{(\alpha(\overline{x},t)+e^{i\varphi(\eta)}\beta(\overline{x},t))}{\sqrt{2}}$

$$\rho_s = \frac{1}{2}\begin{pmatrix} |\alpha(\bar{x},t)|^2 & \alpha^*(\bar{x},t)\beta(\bar{x},t)\mathbb{E}[e^{i\varphi(\eta)}] \\ \beta^*(\bar{x},t)\alpha(\bar{x},t)\mathbb{E}[e^{-i\varphi(\eta)}] & |\beta(\bar{x},t)|^2 \end{pmatrix}$$

Uncorrelated events: $\mathbb{E}[H_{int}(\overline{x}(t))H_{int}(\overline{x}(t'))] \to 0$

$$\mathbb{E}[e^{i\varphi}] = e^{i\mathbb{E}[\varphi]-\frac{1}{2}\mathbb{E}[\delta\varphi^2]} \qquad \delta\varphi = \varphi - \mathbb{E}[\varphi]$$

Literature and open issues



Gravitational decoherence of atomic interferometers

Brahim Lamine[1], Marc-Thierry Jaekel[2], and Serge Reynaud[1]

[1] Laboratoire Kastler Brossel [a], Université Pierre et Marie Curie, case 74, Campus Jussieu, place Jussieu, F-75252 Paris Cedex 05
[2] Laboratoire de Physique Théorique [b], École Normale Supérieure, 24 rue Lhomond, F-75231 Paris Cedex 05

Received: February 14, 2002

Abstract. We study the decoherence of atomic interferometers due to the scatt… tional waves. We evaluate the 'direct' gravitational effect registered by the ph…

Metric fluctuations and decoherence

Heinz-Peter Breuer[1,2], Ertan Göklü[3] and Claus Lämmerzahl[3]

[1] Physikalisches Institut, Universität Freiburg, Hermann-Herder-Strasse 3, 79104 Freiburg, Germany
[2] Hanse–Wissenschaftskolleg, Institute for Advanced Study, 27753 Delmenhorst, Germany
[3] ZARM, University of Bremen, Am Fallturm, 28359 Bremen, Germany

February 25, 2013

Abstract

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A Master Equation for Gravitational Decoherence: Probing the Textures of Spacetime

C. Anastopoulos[1] and B. L. Hu[2]

[1] Department of Physics, University of Patras, 26500 P…
[2] Maryland Center for Fundamental Physics and Joint … University of Maryland, College Park, Maryland 20742

E-mail: anastop@physics.upatras.gr, blhu@umd.edu

Abstract. We give a first principles derivation of a m… of a quantum matter field in a linearly perturbed Minko… quantum field theory and general relativity. We make … introduce extra ingredients, as is often done in altern… the quantum matter field is projected to a one-particle … non-relativistic quantum particle in a weak gravitation… the *energy basis*, in contrast to most existing theorie… We point out the gauge nature of time and space … gravity couplings, and warn that 'intrinsic' decoherence…

PROCEEDINGS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Decoherence of quantum wave packets due to interaction with conformal space –time fluctuations

W. L. Power and I. C. Percival

Proc. R. Soc. Lond. A 2000 **456**, doi: 10.1098/rspa.2000.0544

- **Different predictions**: decoherence in position vs momentum

Literature and open issues

Gravitational decoherence of atomic interferometers

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PACS. 04.30.-w

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model of metric fluctuations has been proposed which yields an effective Schrö- n for a quantum particle with a modified inertial mass, leading to a violation of valence principle. The renormalization of the inertial mass tensor results from a rage over the fluctuations of the metric over a fixed background metric. Here we

A Master Equation for Gravitational Decoherence: Probing the Textures of Spacetime

C. Anastopoulos[1] and B. L. Hu[2]
[1]Department of Physics, University of Patras, 26500 P
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E-mail: anastop@physics.upatras.gr, blhu@umd.edu

Abstract. We give a first principles derivation of a m of a quantum matter field in a linearly perturbed Minko quantum field theory and general relativity. We make introduce extra ingredients, as is often done in altern the quantum matter field is projected to a one-particle non-relativistic quantum particle in a weak gravitation the *energy basis*, in contrast to most existing theorie We point out the gauge nature of time and space gravity couplings, and warn that 'intrinsic' decoherence

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- Scalar bosonic matter only
- Non lab gauge choices
- Assumptions on the size on GWs (short or long waves)
- Brute force non relativistic limit
- **Different predictions**: decoherence in position vs momentum

Our model

Gravitational decoherence: a general non relativistic model

L. Asprea,[1,2,*] G. Gasbarri,[3,2,†] and A. Bassi[1,2,‡]

[1] *Department of Physics, University of Trieste, Strada Costiera 11, 34151 Trieste, Italy*

[2] *Istituto Nazionale di Fisica Nucleare, Trieste Section, Via Valerio 2, 34127 Trieste, Italy*

[3] *Department of Physics and Astronomy, University of Southampton, Highfield Campus, SO17 1BJ, United Kingdom*

(Dated: May 3, 2019)

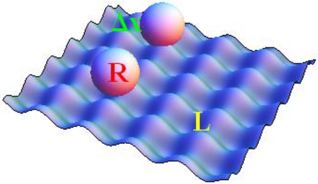
arXiv:1905.01121 [quant-ph]

- Deal with scalar bosons
- Expand the action around flat spacetime to get the EOM assuming no effect on measuring device (lab gauge choice)
- No unnecessary preliminary assumptions on the size of the GWs
- Non relativistic limit: Foldy-Wouthuysen transformation

Single particle Hamiltonian

$$\hat{H} \simeq \frac{\hat{\mathbf{p}}^2}{2m} + mc^2 \frac{h_{00}(t,\hat{\mathbf{x}})}{2} + \frac{1}{2} h_{00}(t,\hat{\mathbf{x}}) \frac{\hat{\mathbf{p}}^2}{2m} + \frac{1}{2m} h_{ij}(t,\hat{\mathbf{x}}) \hat{p}^i \hat{p}^j + c h^{0i}(t,\hat{\mathbf{x}}) \hat{p}_i$$

Extended rigid body



- Assume perfectly rigid body of N paticles
- $\hat{m}(\mathbf{x}) \;=\; \sum_i \frac{m_i}{(2\pi\hbar)^3} \int d\mathbf{q}\, e^{-\frac{i}{\hbar}(\mathbf{x}-\hat{\mathbf{x}}_\mathbf{i})\cdot\mathbf{q}}$
- Assume $h^{\mu\nu}$ does not excite the rotational d.o.f.

Rigid body's center of mass Hamiltonian

$$\hat{H} = Mc^2 + \frac{\hat{\mathbf{P}}^2}{2M} + \int d^3r\, h^{00}(\mathbf{r},t) m(\hat{\mathbf{X}}+\mathbf{r})c^2 +$$

$$- \int d^3r\, h^{00}(\mathbf{r},t)\frac{m(\mathbf{r}+\hat{\mathbf{X}})}{M}\frac{\hat{\mathbf{P}}^2}{4M} + c\int d^3r\, h^{0i}(\mathbf{r},t)\frac{m(\mathbf{r}+\hat{\mathbf{X}})}{M}\hat{P}_i +$$

$$- \int d^3r\, h^{ij}(\mathbf{r},t)\frac{m(\mathbf{r}+\hat{\mathbf{X}})}{M}\frac{\hat{P}_i\hat{P}_j}{2M}$$

Stochastic perturbation: master equation

- $\mathbb{E}[h_{\mu\nu}(\mathbf{x},t)] = 0 \qquad \mathbb{E}[h_{\mu\nu}(\mathbf{x},t)h_{\nu\rho}(\mathbf{y},s)] = \alpha^2 f_{\mu\rho}(\mathbf{x},\mathbf{y};t,s)$

General (non markovian) master equation

$$\partial_t\hat{\rho} = -\frac{i}{\hbar}[\hat{H}_0,\hat{\rho}(t)] - \frac{\alpha^2}{\hbar^8}\int\frac{d^3q\,d^3q'}{(2\pi)^3}\int_0^t dt_1\,\tilde{f}^{00}(\mathbf{q},\mathbf{q}';t,t_1)\frac{m(\mathbf{q})m(\mathbf{q}')}{M^2}\cdot$$

$$\cdot\left[e^{i\mathbf{q}\cdot\hat{\mathbf{x}}/\hbar}\left(\frac{\hat{P}^2}{4M}+\frac{Mc^2}{2}\right),\left[e^{i\mathbf{q}'\cdot\hat{\mathbf{x}}_{t_1}/\hbar}\left(\frac{\hat{P}^2}{4M}+\frac{Mc^2}{2}\right),\hat{\rho}(t)\right]\right]+$$

$$-\frac{\alpha^2c^2}{\hbar^8}\int\frac{d^3q\,d^3q'}{(2\pi)^3}\int_0^t dt_1\,\tilde{f}^{0i}(\mathbf{q},\mathbf{q}';t,t_1)\frac{m(\mathbf{q})m(\mathbf{q}')}{M^2}\left[e^{i\mathbf{q}\cdot\hat{\mathbf{x}}/\hbar}\hat{P}_i,\left[e^{i\mathbf{q}'\cdot\hat{\mathbf{x}}_{t_1}/\hbar}\hat{P}_i,\hat{\rho}(t)\right]\right]+$$

$$-\frac{\alpha^2}{\hbar^8}\int\frac{d^3q\,d^3q'}{(2\pi)^3}\int_0^t dt_1\,\tilde{f}^{ij}(\mathbf{q},\mathbf{q}';t,t_1)\frac{m(\mathbf{q})m(\mathbf{q}')}{M^2}\left[e^{i\mathbf{q}\cdot\hat{\mathbf{x}}/\hbar}\frac{\hat{P}_i\hat{P}_j}{2M},\left[e^{i\mathbf{q}'\cdot\hat{\mathbf{x}}_{t_1}/\hbar}\frac{\hat{P}_i\hat{P}_j}{2M},\hat{\rho}(t)\right]\right]+$$

$$+O(t\alpha^3\tau_c^2)$$

Markovian limit

- Assume roto-translational invariance of correlation functions: $f^{\mu\nu}(\mathbf{x},\mathbf{y};t,s) = \frac{L}{c}u^{\mu\nu}(\mathbf{x}-\mathbf{y})\delta(t-s)$

Markovian master equation

$$\partial_t\hat{\rho} = -\frac{i}{\hbar}[\hat{H}_0,\hat{\rho}(t)] - \frac{\alpha^2 Lc^3}{4(2\pi)^{3/2}\hbar^5}\int d^3q\,\tilde{u}^{00}(\mathbf{q})m^2(\mathbf{q})\left[e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar},\left[e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar},\hat{\rho}(t)\right]\right]+$$

$$-\frac{\alpha^2 L}{(2\pi)^{3/2}\hbar^5 c}\int d^3q\,\tilde{u}^{00}(\mathbf{q})\frac{m^2(\mathbf{q})}{M^2}\left[e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}\frac{\hat{\mathbf{P}}^2}{2M},\left[e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}\frac{\hat{\mathbf{P}}^2}{2M},\hat{\rho}(t)\right]\right]+$$

$$-\frac{\alpha^2 Lc}{2(2\pi)^{3/2}\hbar^5}\int d^3q\,\tilde{u}^{00}(\mathbf{q})\frac{m^2(\mathbf{q})}{M}\left[e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar},\left[e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}\frac{\hat{\mathbf{P}}^2}{2M},\hat{\rho}(t)\right]\right]+$$

$$-\frac{\alpha^2 Lc}{2(2\pi)^{3/2}\hbar^5}\int d^3q\,\tilde{u}^{00}(\mathbf{q})\frac{m^2(\mathbf{q})}{M}\left[e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}\frac{\hat{\mathbf{P}}^2}{2M},\left[e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar},\hat{\rho}(t)\right]\right]+$$

$$-\frac{\alpha^2 Lc}{(2\pi)^{3/2}\hbar^5}\int d^3q\,\tilde{u}^{0i}(\mathbf{q})\frac{m^2(\mathbf{q})}{M^2}\left[e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}\hat{P}_i,\left[e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}\hat{P}_i,\hat{\rho}(t)\right]\right]+$$

$$-\frac{\alpha^2 L}{(2\pi)^{3/2}\hbar^5 c}\int d^3q\,\tilde{u}^{ij}(\mathbf{q})\frac{m^2(\mathbf{q})}{M^2}\left[e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}\frac{\hat{P}_i\hat{P}_j}{2M},\left[e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}\frac{\hat{P}_i\hat{P}_j}{2M},\hat{\rho}(t)\right]\right]$$

Decoherence in position

Dominant contribution when: $h^{00} \gtrsim h^{0i}, h^{ij}$

Master equation with decoherence in position

$$\partial_t \hat{\rho} \simeq -\frac{i}{\hbar}[\hat{H}_0, \hat{\rho}(t)] - \frac{\alpha^2 L c^3}{(2\pi)^{3/2}\hbar^5} \int d^3q \, \tilde{u}^{00}(\mathbf{q}) m^2(\mathbf{q}) \left[e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \left[e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \hat{\rho}(t) \right] \right]$$

- It recovers* the work of Blencowe:

$$u^{00}(\mathbf{x}-\mathbf{x}') = L^3 \delta^3(\mathbf{x}-\mathbf{x}')$$

- It recovers the work of Sanchez Gomez and Power-Percival:

$$m(r) = M\delta^3(r) \qquad \tilde{u}^{00}(\mathbf{q}-\mathbf{q}') = L^3\hbar^3\delta(\mathbf{q}-\mathbf{q}')e^{-\hbar^2\mathbf{q}^2L^2/2}$$

Decoherence in momentum

Dominant contribution when $e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \sim \hat{\mathbb{I}}$

Master equation with decoherence in momentum

$$\partial_t \hat{\rho} = -\frac{i}{\hbar}[\hat{H}_0, \hat{\rho}(t)] - \frac{\alpha^2 L}{(2\pi)^{3/2}\hbar^5 c} \int d^3q\, \tilde{u}^{00}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M^2} \left[\frac{\hat{\mathbf{P}}^2}{2M}, \left[\frac{\hat{\mathbf{P}}^2}{2M}, \hat{\rho}(t)\right]\right]$$
$$- \frac{\alpha^2 Lc}{(2\pi)^{3/2}\hbar^5} \int d^3q\, \tilde{u}^{0i}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M^2} \left[\hat{P}_i, \left[\hat{P}_i, \hat{\rho}(t)\right]\right]$$
$$- \frac{\alpha^2 L}{(2\pi)^{3/2}\hbar^5 c} \int d^3q\, \tilde{u}^{ij}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M^2} \left[\frac{\hat{P}_i\hat{P}_j}{2M}, \left[\frac{\hat{P}_i\hat{P}_j}{2M}, \hat{\rho}(t)\right]\right]$$

- It recovers the work of Breuer et al. and Anastopoulos-Hu*:

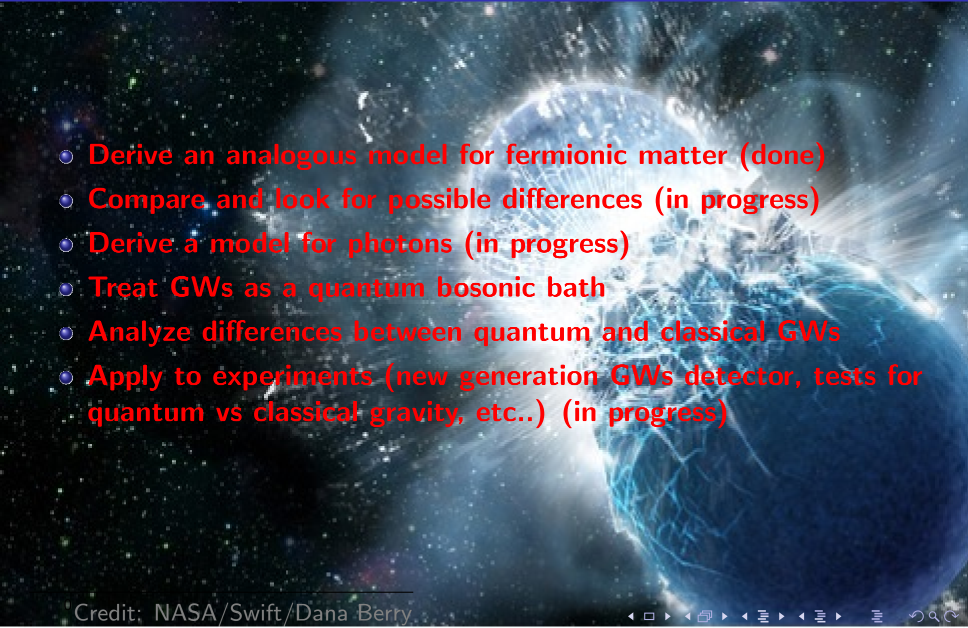
$$m(\mathbf{r}) = \frac{m}{(\sqrt{2\pi}R)^3} e^{-\mathbf{r}^2/(2R^2)} \qquad h^{ij} \gg h^{0i}, h^{00}$$

$$\tilde{u}^{ij}(\mathbf{q}-\mathbf{q}') = \delta^{ij} L^3 \hbar^3 \delta(\mathbf{q}-\mathbf{q}') e^{-\hbar^2\mathbf{q}^2 L^2/2}$$

Conclusions

- A gravitational perturbation induces a phase accumulation in a quantum system
- This phase accumulation is responsible for a decoherence effect
- The models describing the phenomenon found in the literature refer to different regimes of approximations and have different predictions
- Our model predicts decoherence in both position and momentum eigenbasis
- Our model is able to recover all of the results (that we were aware of) present in the literature when the appropriate limit is taken
- Different preferred basis puzzle solved

To be done



- Derive an analogous model for fermionic matter (done)
- Compare and look for possible differences (in progress)
- Derive a model for photons (in progress)
- Treat GWs as a quantum bosonic bath
- Analyze differences between quantum and classical GWs
- Apply to experiments (new generation GWs detector, tests for quantum vs classical gravity, etc..) (in progress)

Credit: NASA/Swift/Dana Berry

THANK YOU!

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arXiv:1905.01121 [quant-ph] (2019)

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Credit: The SXS Project