Entanglement Distribution via Separable States

& Incoherent Dynamics

Hannah McAleese

QUEEN'S UNIVERSITY BELFAST

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QUANTUM TECHNOLOGY at QUEEN'S

Outline

1 Entanglement Distribution

- Quantum communication
- ED with Separable States

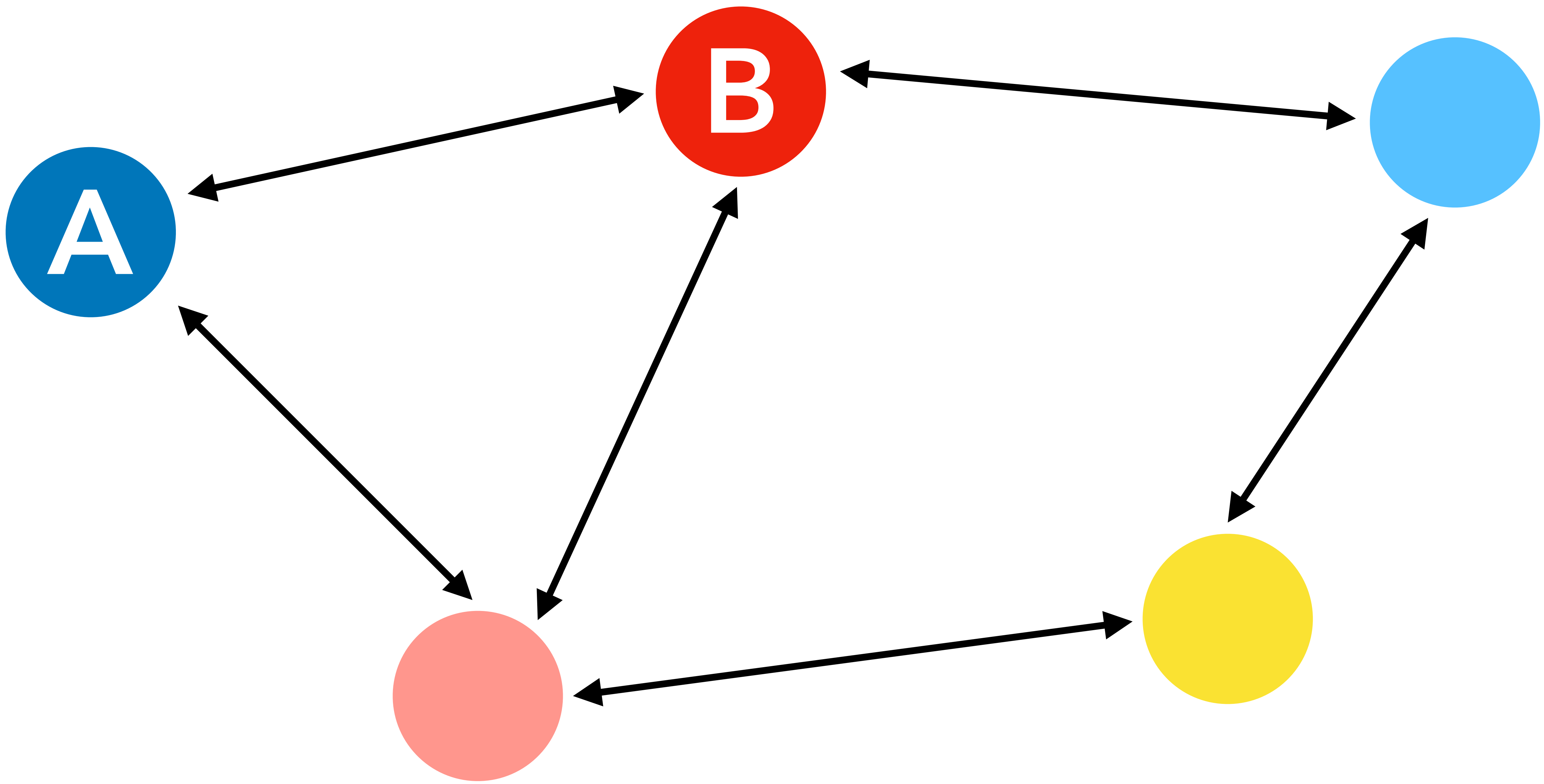
2 Incoherent Dynamics

- Motivation
- Our approach

3 Results

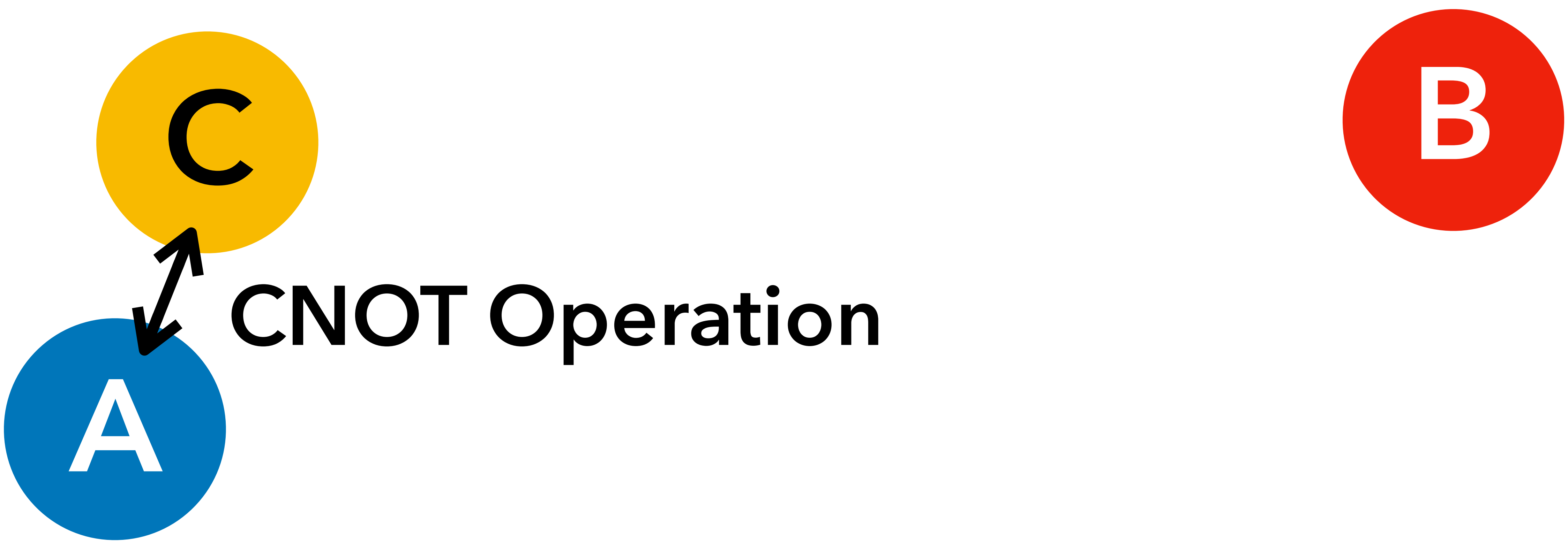
Quantum Communication

The Goal: Quantum Internet



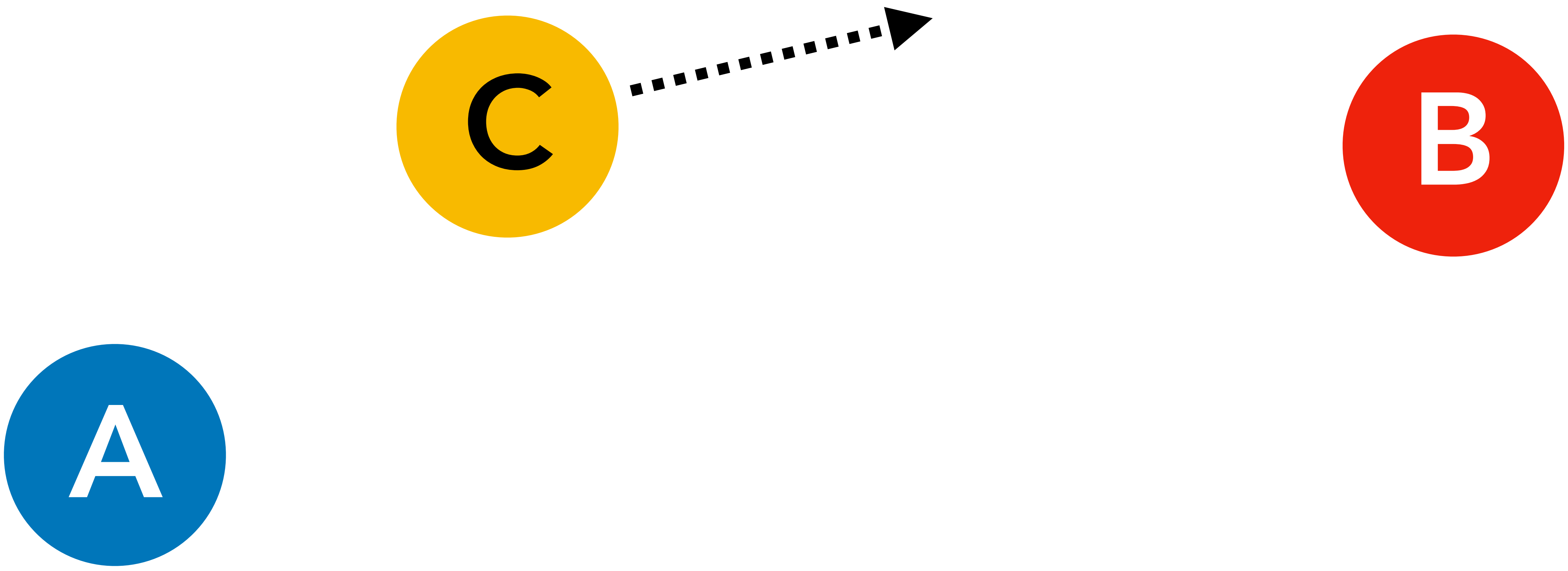
Entanglement Distribution

1 Encoding Operation



Entanglement Distribution

2 Send C from A to B

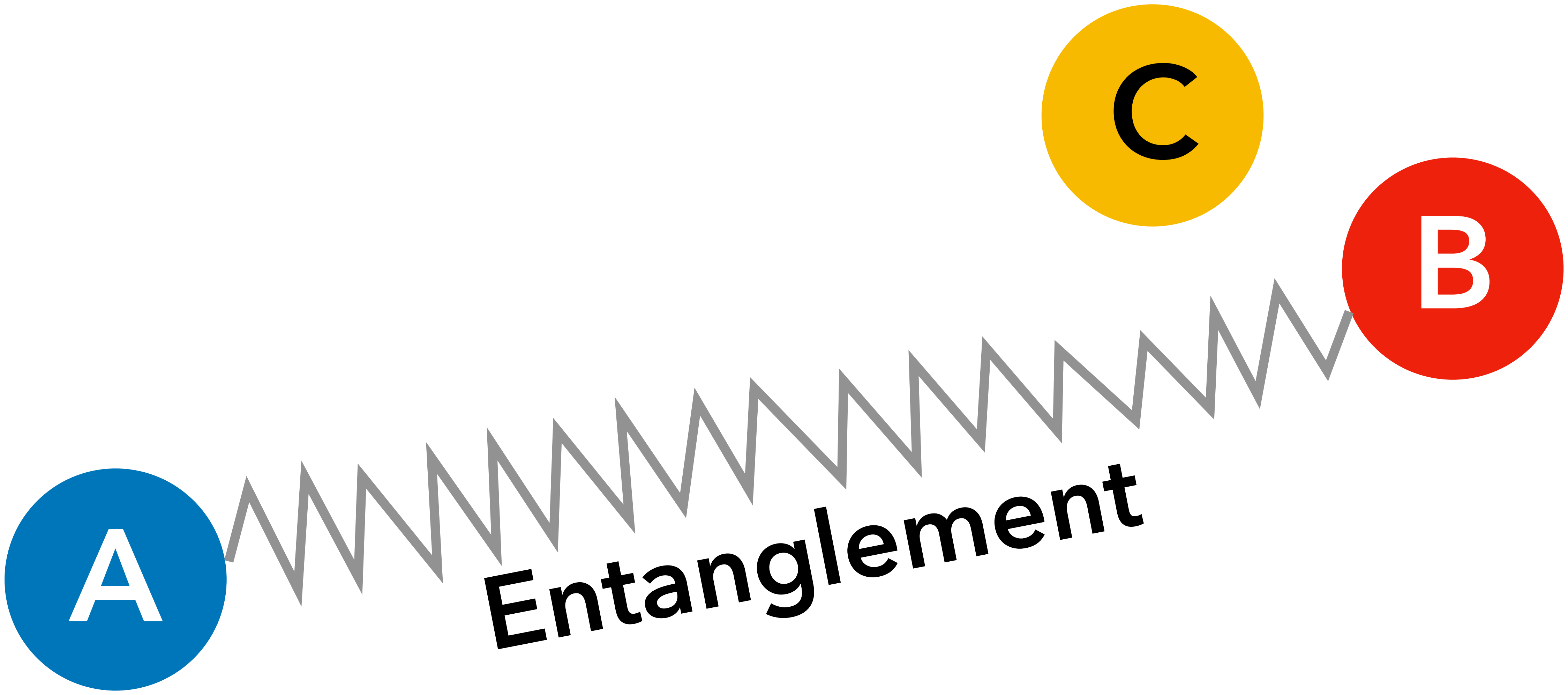


Entanglement Distribution

3 Decoding Operation

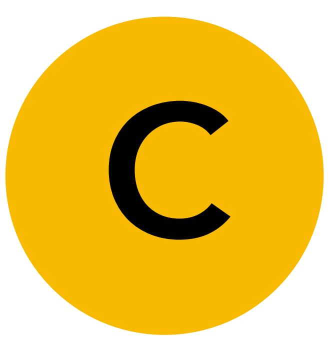
Entanglement Distribution

Result: A and B are now entangled



Entanglement Distribution

Result: A and B are now entangled



This result is still possible when the carrier system is always **separable** from A and B

Cubitt et al., PRL (2003)

Quantum Discord

- Mixed states: difference between product states

$$\rho_{AB} = \rho_A \otimes \rho_B$$

with no classical or quantum correlations...

- ...and separable states (no entanglement)

$$\rho_{AB} = \sum_k p_k \rho_A^k \otimes \rho_B^k$$

Quantum Discord

$$\rho_{AB} = \sum_k p_k \rho_A^k \otimes \rho_B^k$$

- Mixed separable states can still have quantum correlations, e.g. discord
- $$D(A\,|\,B) = I(A:B) - J(A\,|\,B)$$

Quantum Discord

$$D(A\,|\,B) = \boxed{I(A:B)} - J(A\,|\,B)$$

- Mutual information
- Measures total correlations (relative entropy between state ρ_{AB} and product state $\rho_A \otimes \rho_B$)

$$I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Quantum Discord

$$D(A|B) = I(A:B) - \boxed{J(A|B)}$$

- Generalised conditional entropy
- Measures classical correlations - maximum info that can be gained about A by measuring B

$$J(A|B) = \max_{B_i^\dagger B_i}\left(S(\rho_A) - \sum_i p_i S(\rho_A^i) \right)$$

Quantum Discord

$$D(A|B) = I(A:B) - J(A|B)$$

- Discord = how much you disturb the overall state when extracting information

- $D(A|B) \neq D(B|A)$
- $D(A|B) \geq 0$

Quantum Discord

In entanglement distribution:

Discord bounds the amount of entanglement gained

Chuan et al., PRL (2012)

$$\varepsilon_{A:CB}(\beta) - \varepsilon_{AC:B}(\alpha) \leq D_{AB|C}(\beta)$$

After sending C   Initial

Entanglement

Incoherent Dynamics

Our work:

What if there are imperfections in the encoding and decoding steps?

$$\rho = U\rho(0)U^\dagger$$

$$\downarrow$$

$$\frac{d\rho}{dt} = -i[H, \rho] + \gamma \mathscr{L}(\rho)$$

Incoherent Dynamics

$$\frac{d\rho}{dt} = -i[H, \rho] + \gamma \mathscr{L}(\rho)$$

Unitary dynamics

H is the Hamiltonian of the CNOT operation: $U_{\text{CNOT}} = e^{-iHt}$

Incoherent dynamics

$$\mathscr{L}_{AC}(\rho) = 2(\sigma_A^+\sigma_C^-)\rho(\sigma_A^-\sigma_C^+) - (\sigma_A^-\sigma_C^+)(\sigma_A^+\sigma_C^-)\rho - \rho(\sigma_A^-\sigma_C^+)(\sigma_A^+\sigma_C^-)$$

$$\sigma^+ = |1\rangle\langle 0|, \sigma^- = |0\rangle\langle 1|$$

Incoherent Dynamics

$$\frac{d\rho}{dt} = -i[H,\rho] + \gamma\mathscr{L}(\rho)$$

Strength of the incoherent dynamics

$\gamma_{AC} \longrightarrow$ **Encoding step**

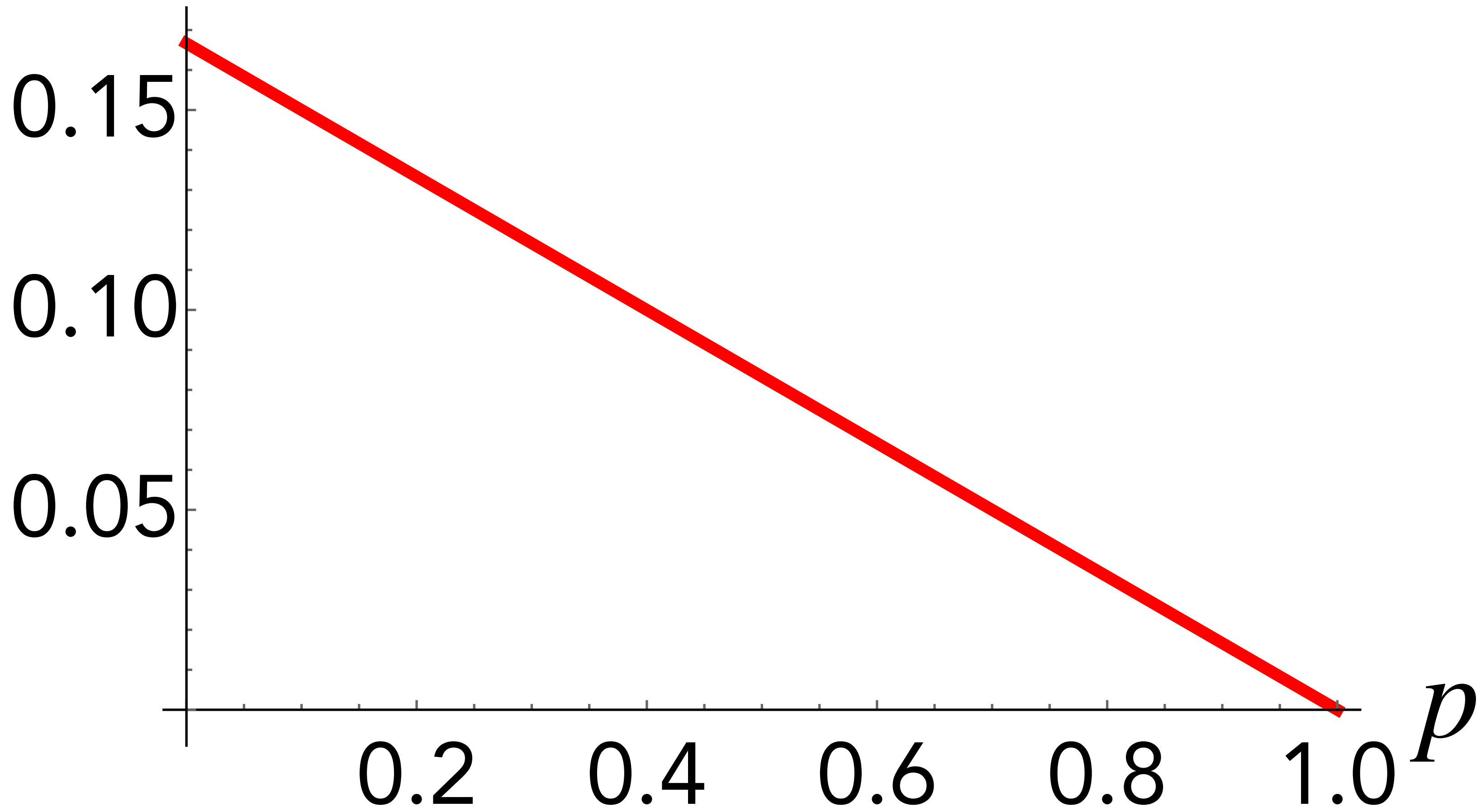
$\gamma_{BC} \longrightarrow$ **Decoding step**

Initial State

$$\alpha(p) = p\Lambda_{\mathrm{sep}} + (1-p)\Lambda_{\mathrm{ent}}$$

Chuan et al., PRL (2012)

Initial Entanglement between A and B



C separable from AB?

How long should the interactions last?

Steady State?

No unique steady state

E.g. for encoding:

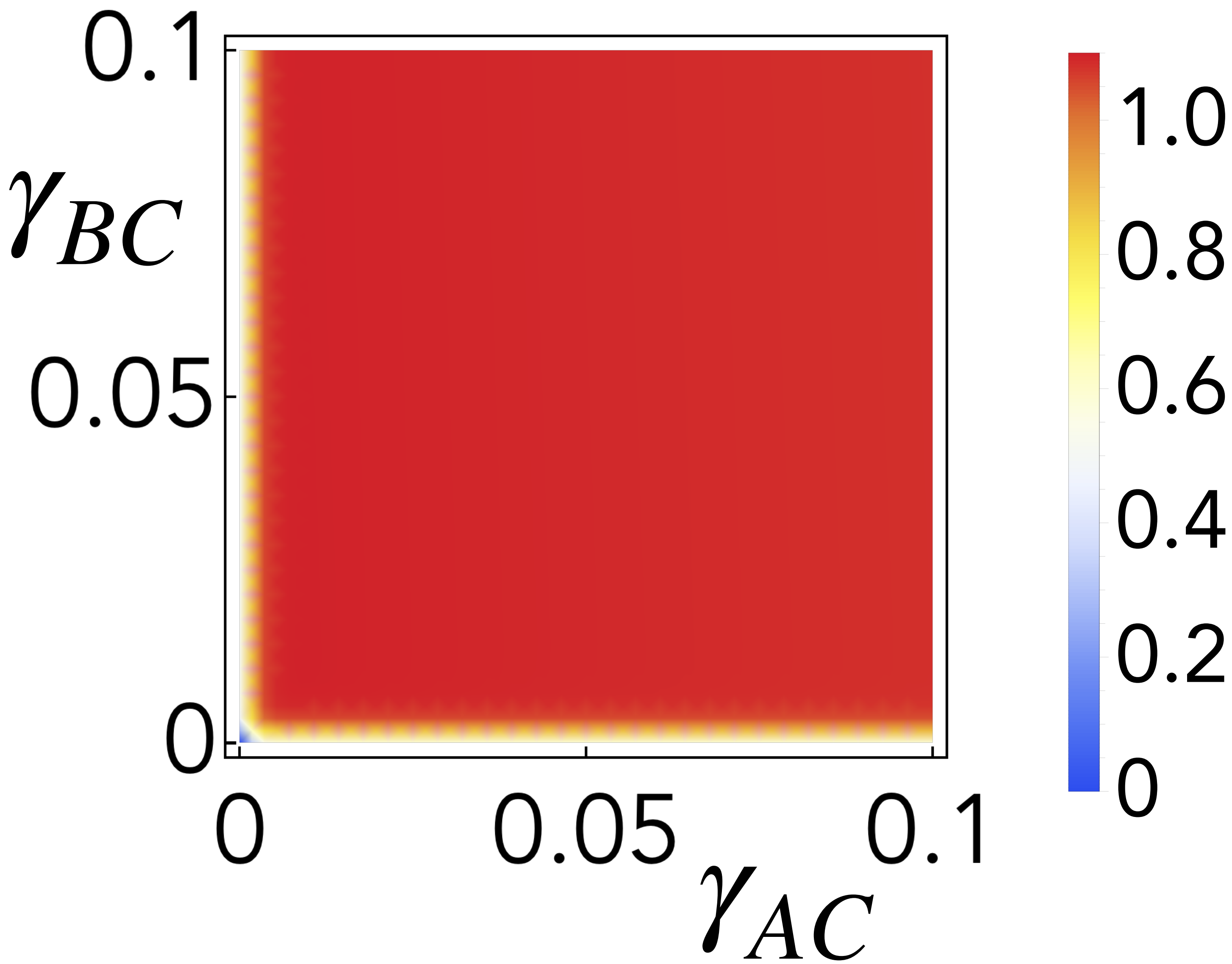
$$\rho = |000\rangle\langle 000| \longrightarrow \frac{d\rho}{dt} = 0$$

$$\rho = |001\rangle\langle 001| \longrightarrow \frac{d\rho}{dt} = 0$$

C separable from AB?

How long should the interactions last?

Steady State?



No steady state where

$$E_{C|AB} = 0$$

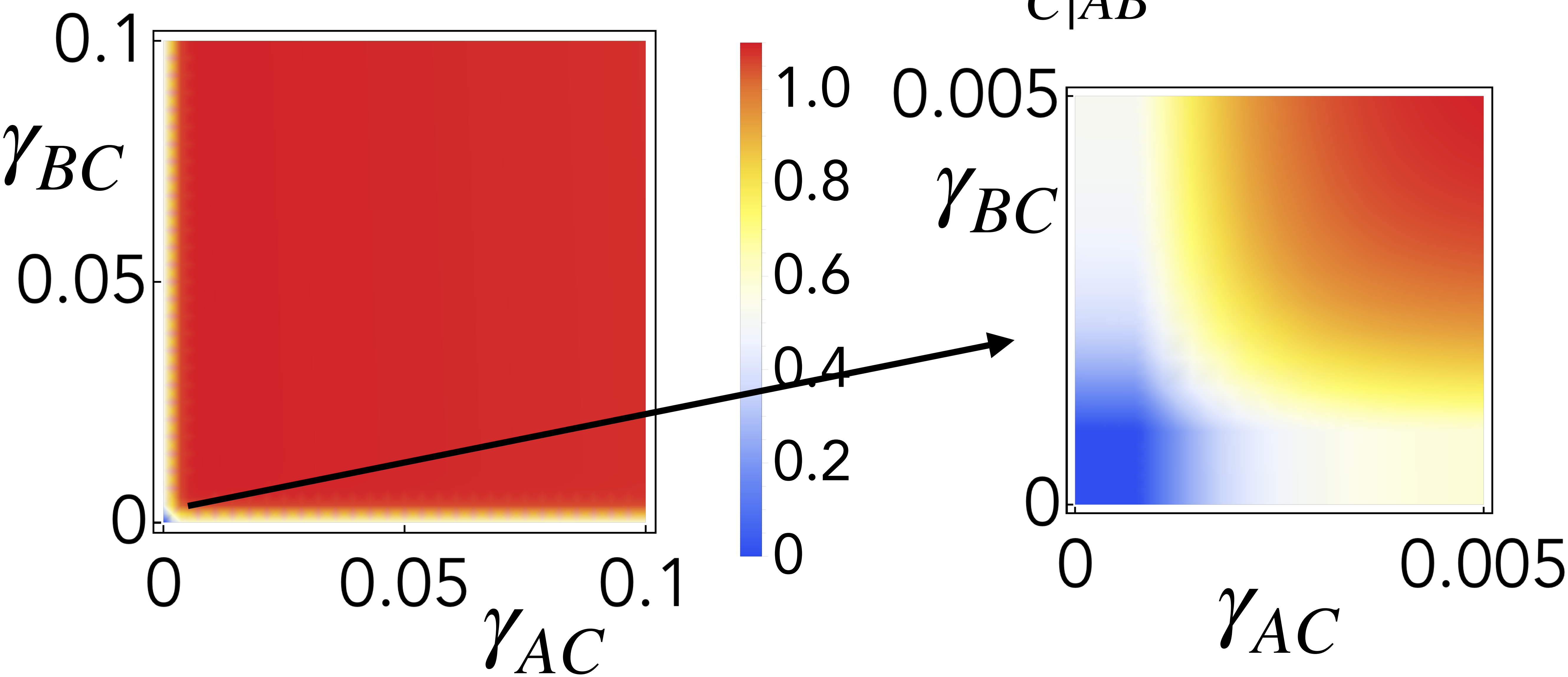
C separable from AB?

How long should the interactions last?

Steady State?

No steady state where

$$E_{C|AB} = 0$$



C separable from AB?

How long should the interactions last?

Steady State?

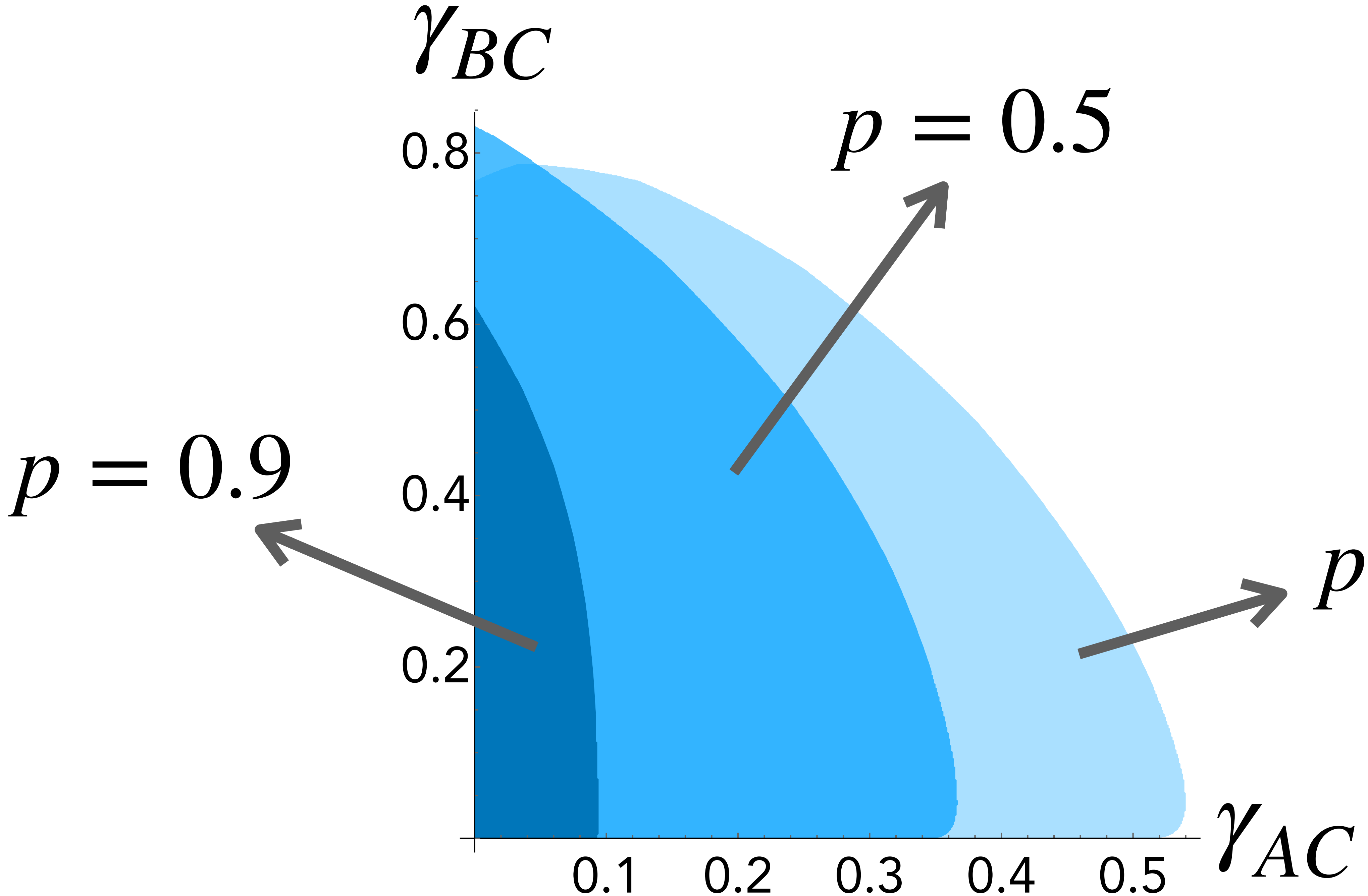
Need to limit interaction time.

Focus on case where

$$0 \leq t \leq 1$$

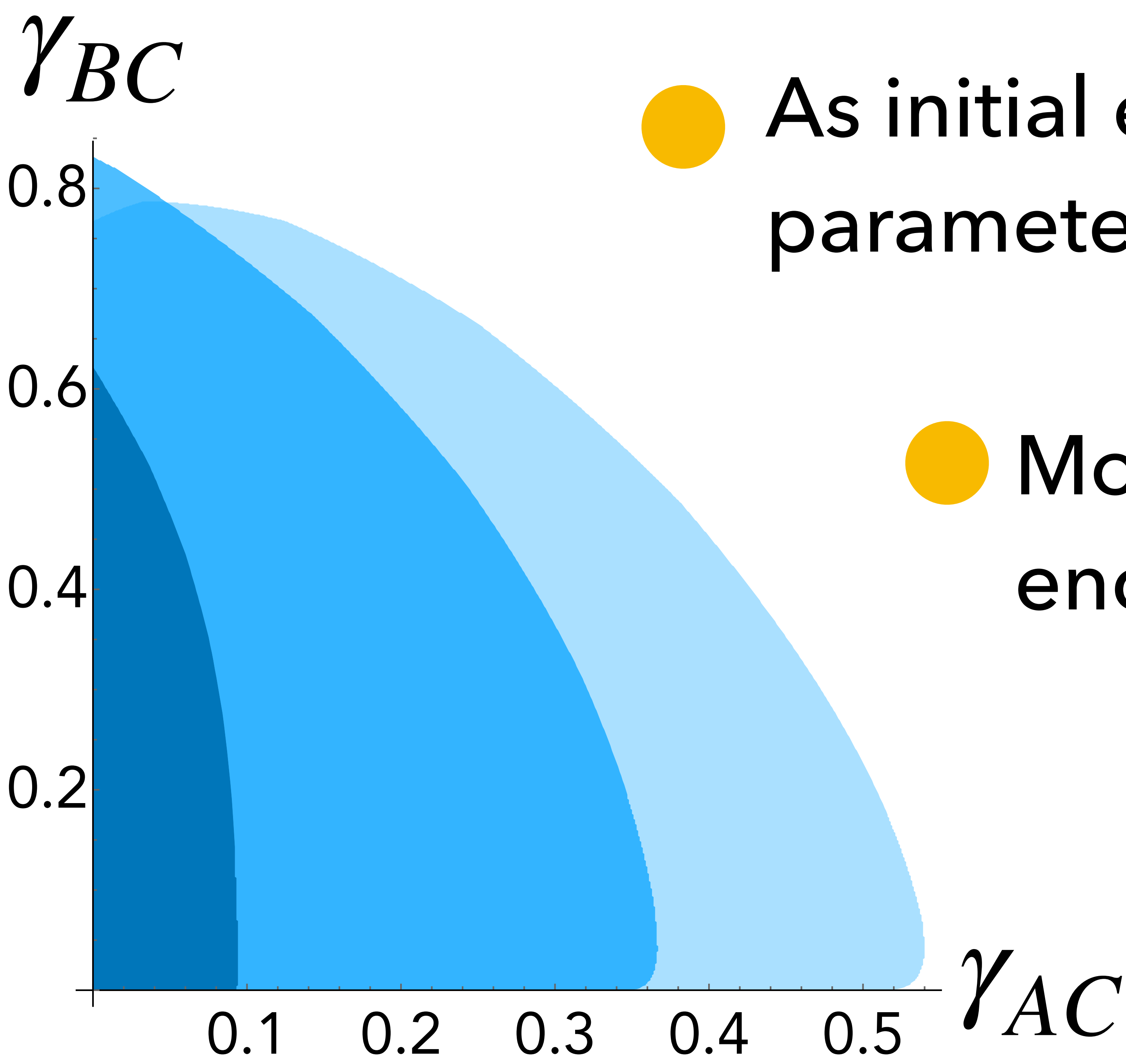
C separable from AB?

Which values of γ_{AC} and γ_{BC} allow for EDSS?



C separable from AB?

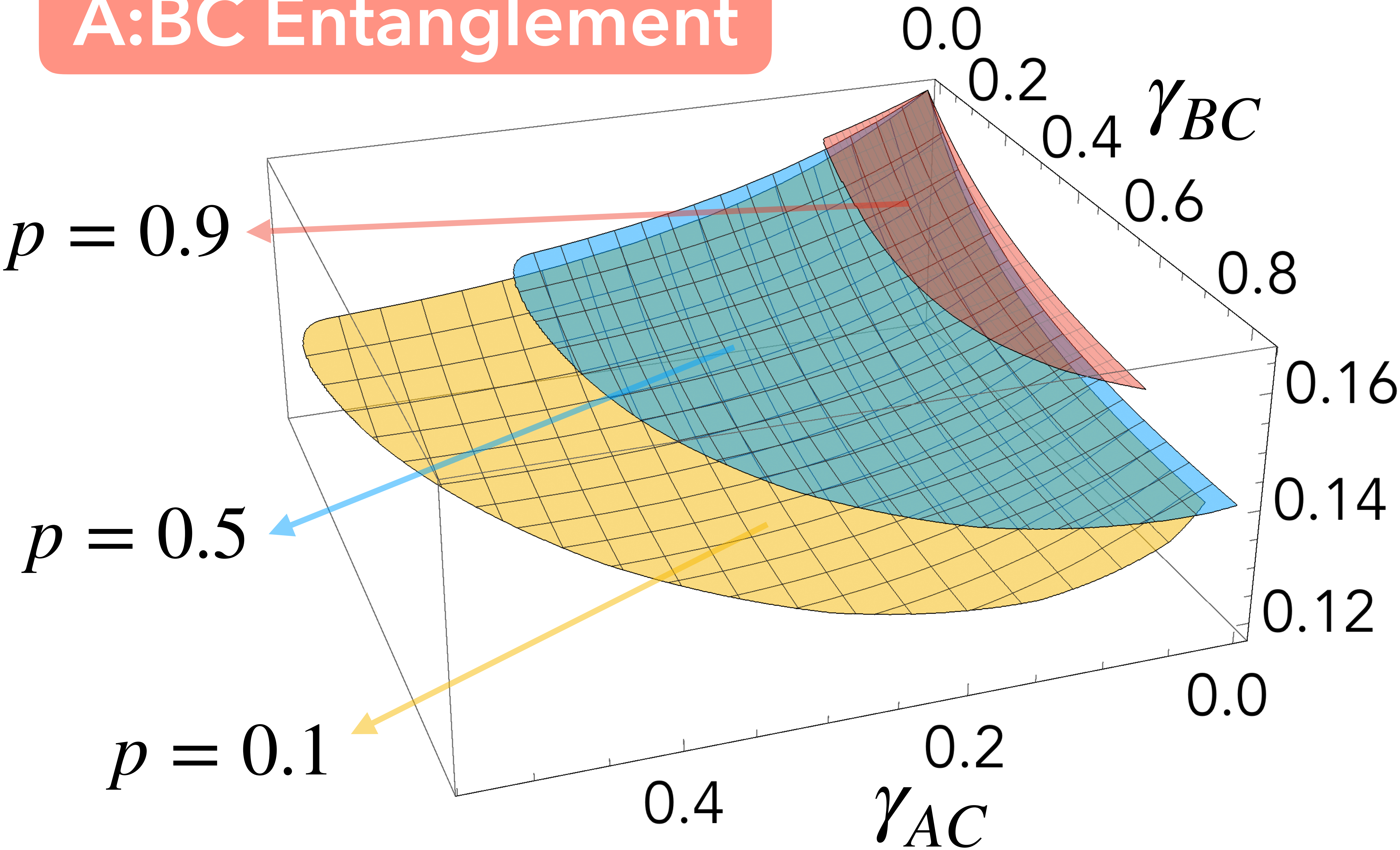
Which values of γ_{AC} and γ_{BC} allow for EDSS?



- As initial entanglement grows, parameters are more restricted
- More restrictions on encoding than decoding

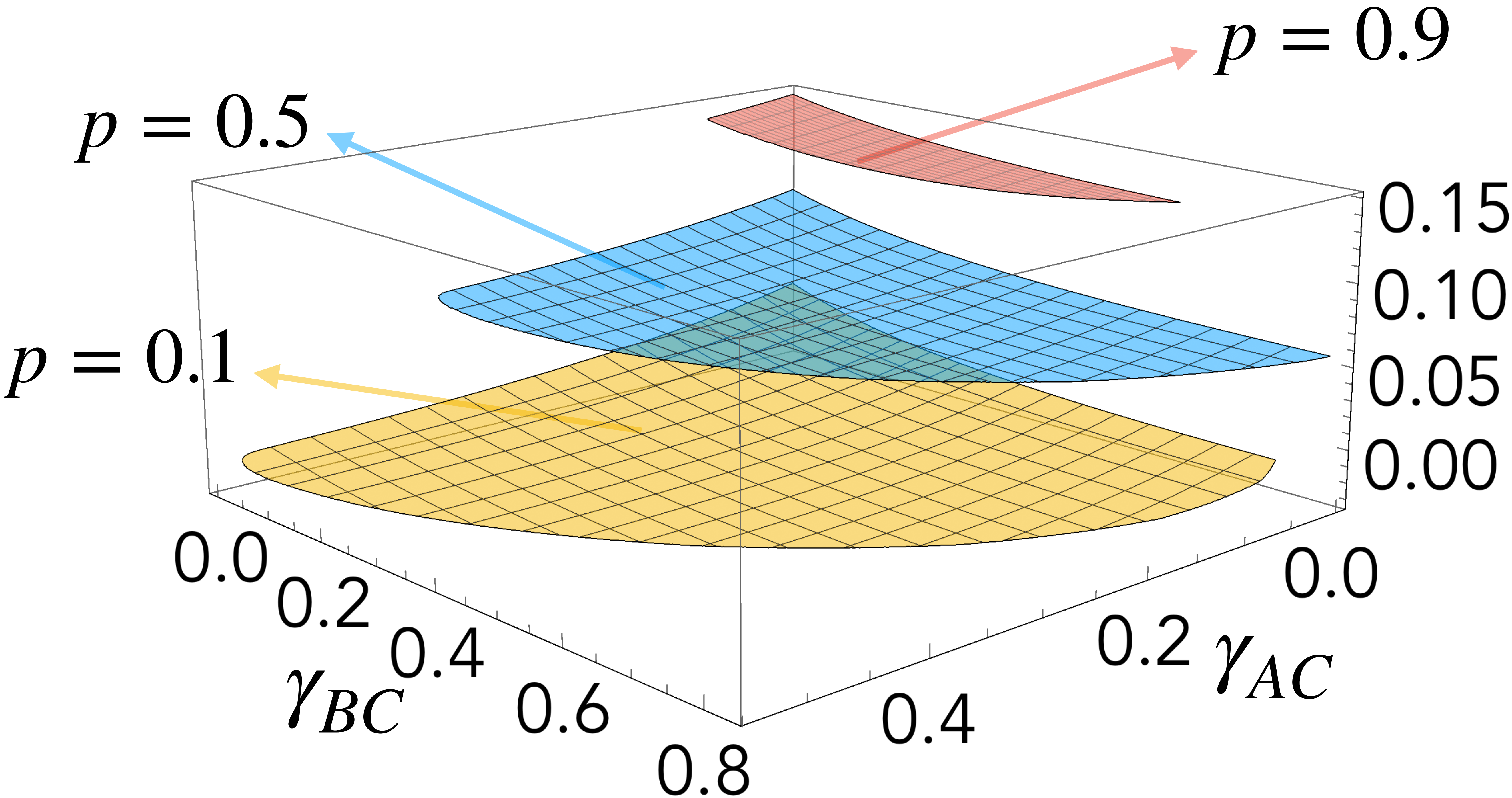
A entangled to BC?

A:BC Entanglement



A entangled to BC?

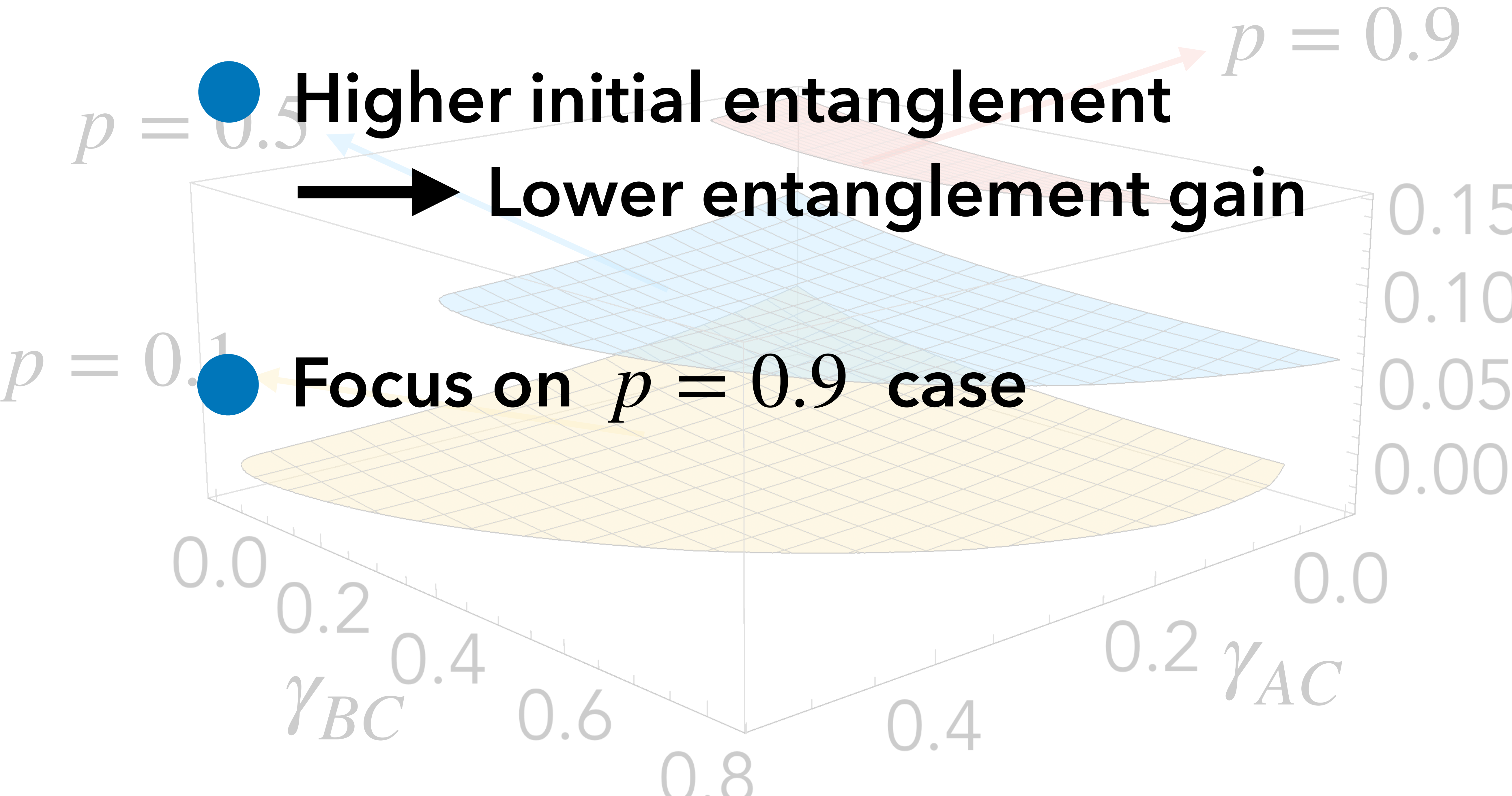
A:BC Entanglement Gain



A entangled to BC?

A:BC Entanglement Gain

- Higher initial entanglement
  → Lower entanglement gain
- Focus on $p = 0.9$ case



A entangled to BC?

A:BC Entanglement Gain

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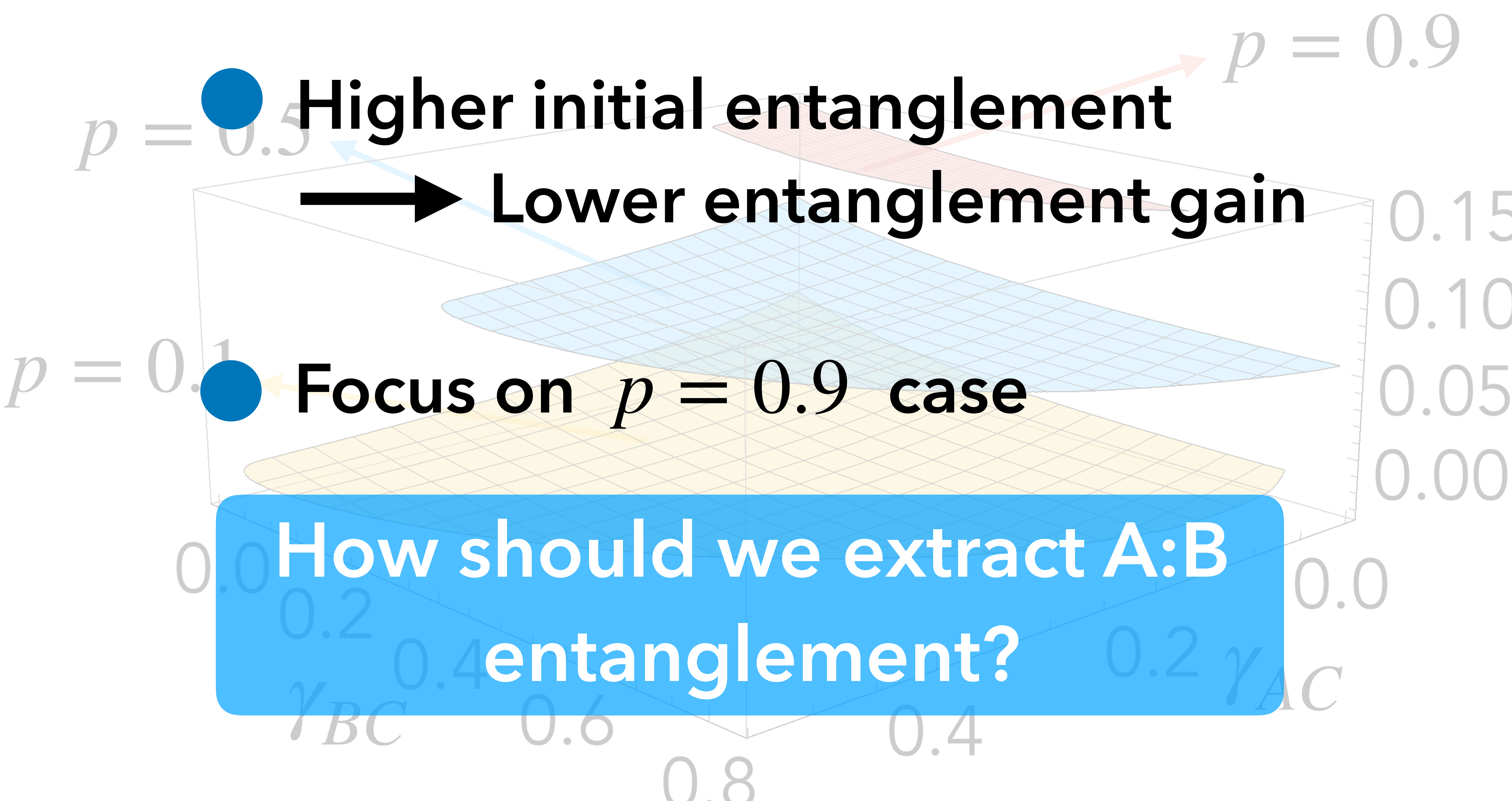


How should we extract A:B entanglement?

A entangled to B?

Measure C

In the case of unitary dynamics, final state:

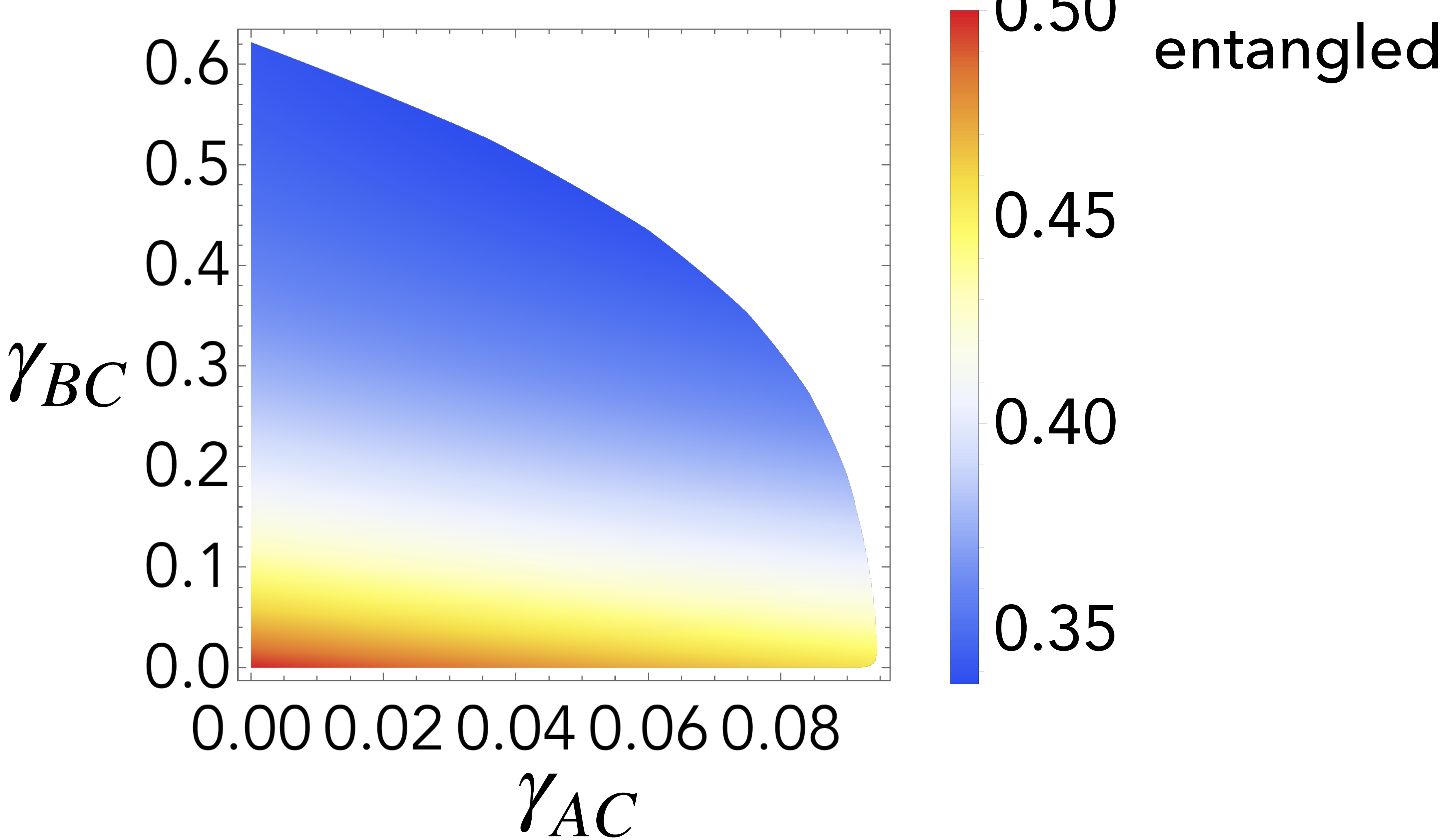
$$\rho_{ABC} = \frac{1}{3} \underbrace{|\phi^+\rangle\langle\phi^+|}_{\text{Bell state - maximally entangled}} \otimes |0\rangle\langle 0| + \frac{2}{3}\sigma^{\text{sep}} \otimes |1\rangle\langle 1|$$

Bell state - maximally entangled

→ Try measuring C in the standard basis

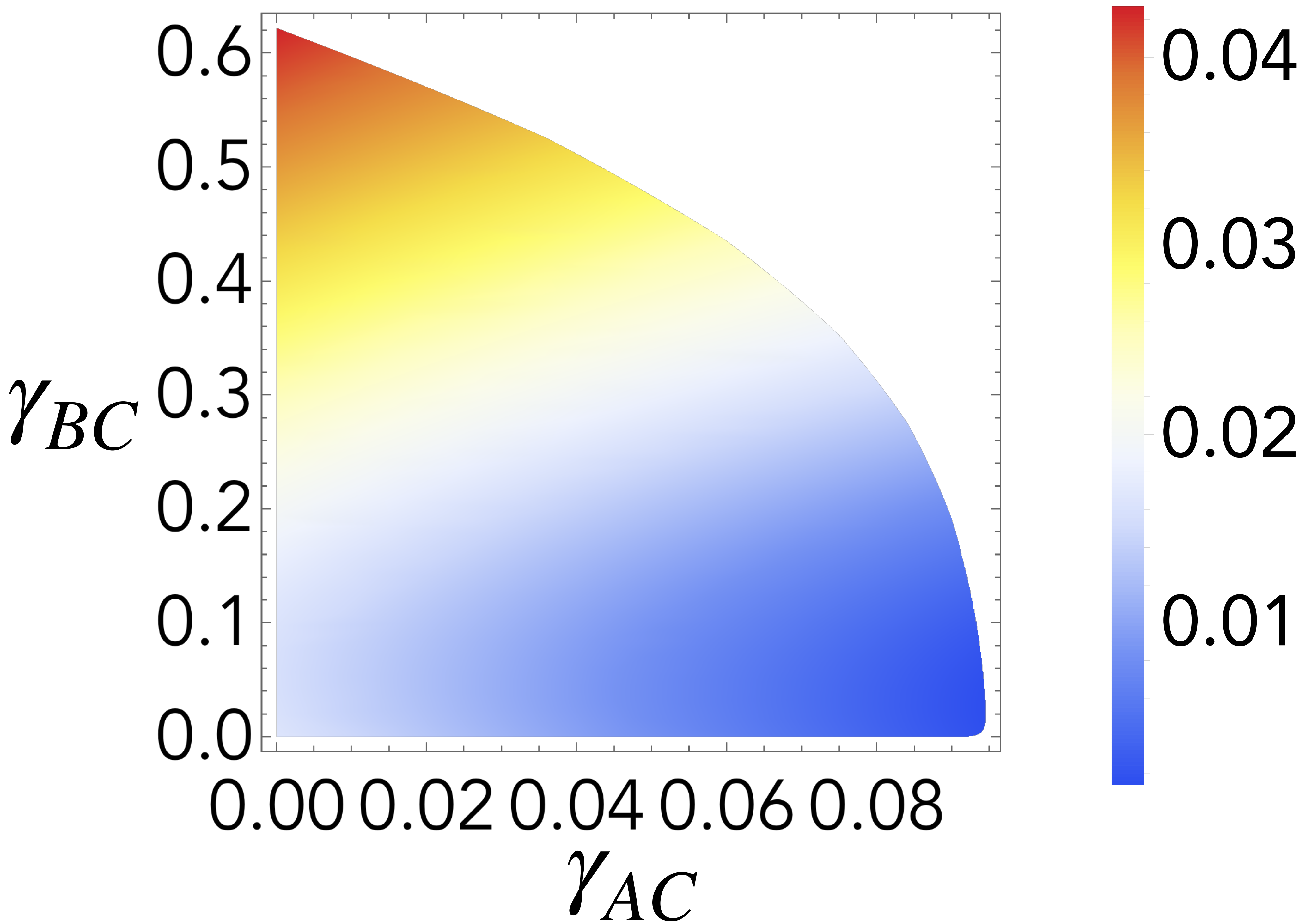
A entangled to B?

Measure C $p = 0.9$



A entangled to B?

Trace out C $p = 0.9$



Conclusions

- It is possible to generate entanglement between 2 systems without using entanglement, **even with incoherent dynamics**

- <u>In this case:</u>

  - There are more restrictions on encoding than decoding

  - Stronger incoherent dynamics can increase A:B entanglement when tracing out C

Thank you:

Prof Mauro Paternostro

Dr Alessandro Ferraro

Department for the Economy
www.economy-ni.gov.uk

Science Foundation Ireland sfi
For what's next

Thank you for your attention!