SISSA

ma per seguir virtute e canoscenza

The Abdus Salam International Centre for Theoretical Physics (ICTP)

Scrambling and entanglements spreading in long range spin chains

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Trieste Junior Quantum Days

A glance in research: where we stand and future challenges

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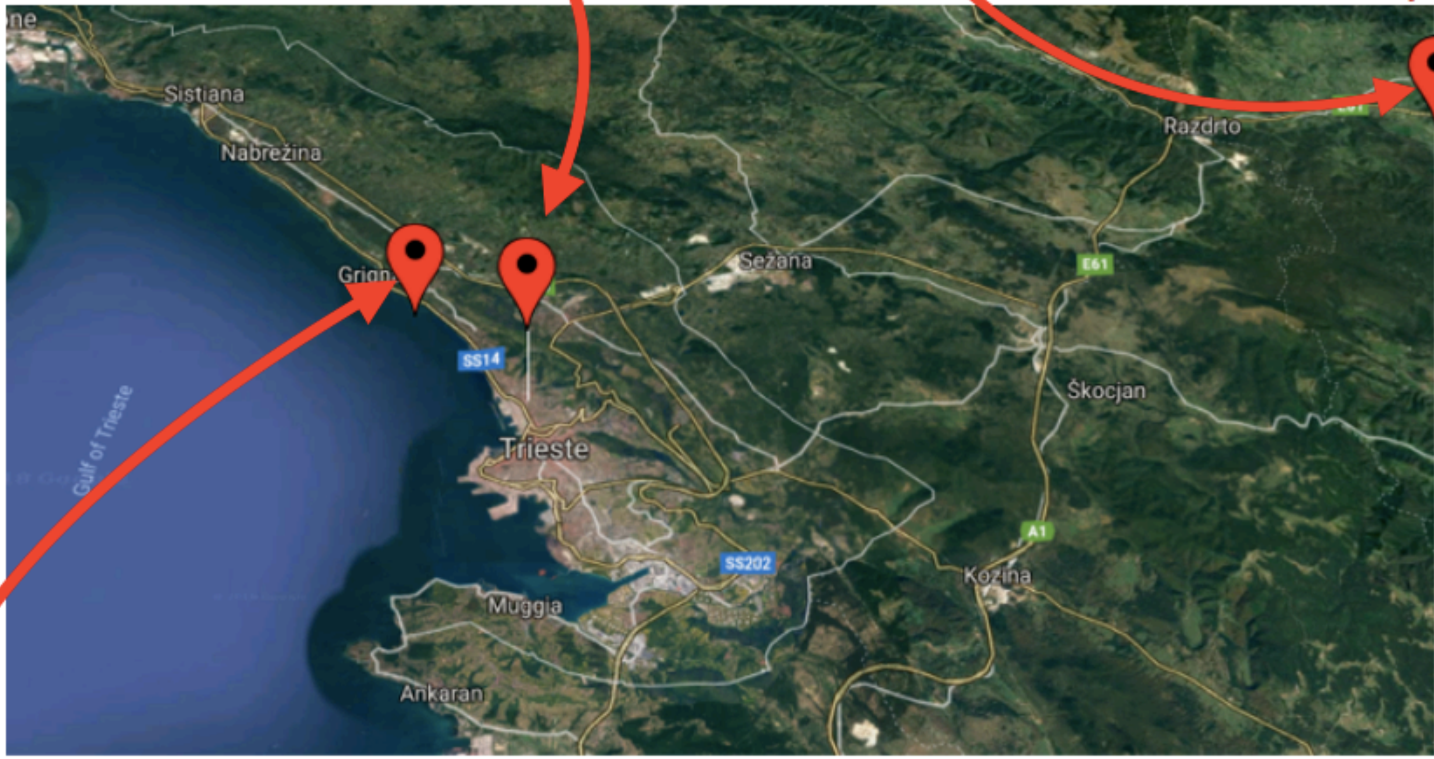


The Abdus Salam
International Centre for Theoretical Physics
ICTP

Rosario Fazio

Angelo Russomanno

Fernando Iemini

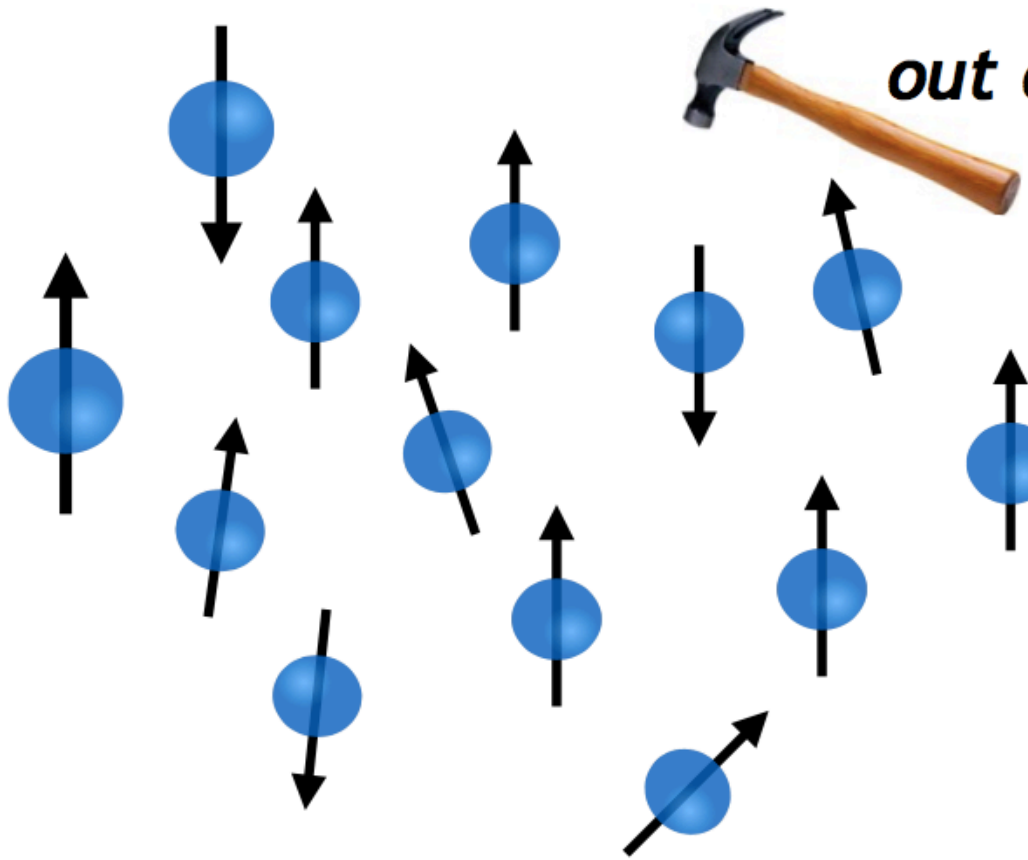
Many-body systems dynamics

$$|\psi_{tot}\rangle \in \bigotimes_{i=1}^{n} \mathcal{H}_i \qquad \text{dim. } 2^n$$

How does information propagates?

$\hat{U}(t) = e^{i\hat{H}t}$ unitary evolution $\qquad |\psi_{tot}(t)\rangle = e^{i\hat{H}t}|\psi_{tot}\rangle$

out of equilibrium



1. "spreading of quantum information across the system"

quantum chaos from the semiclassical limit

⟶ OTOC correlators

Scrambling and entanglements

spreading in long range spin chains

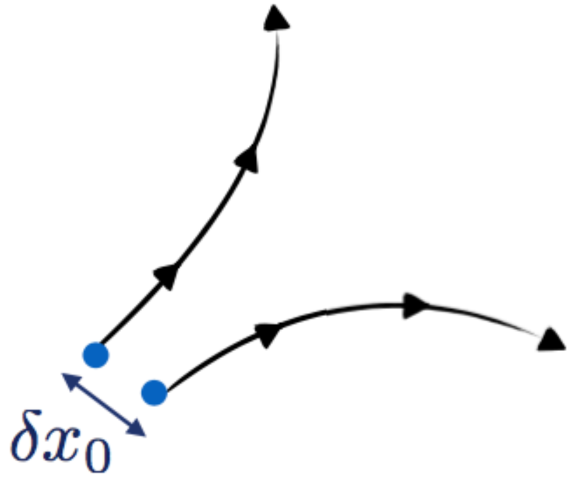
2. ≠

3.

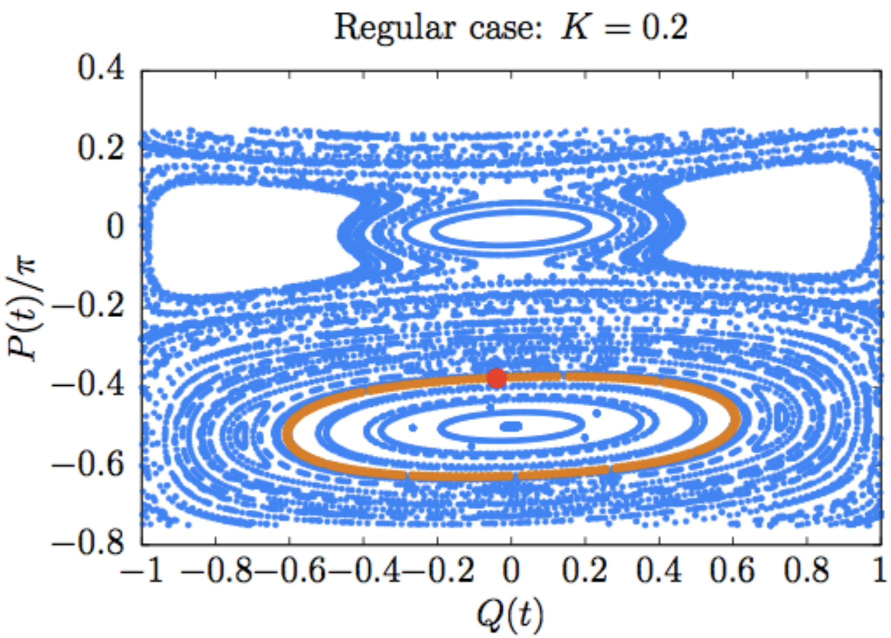
Classical chaos

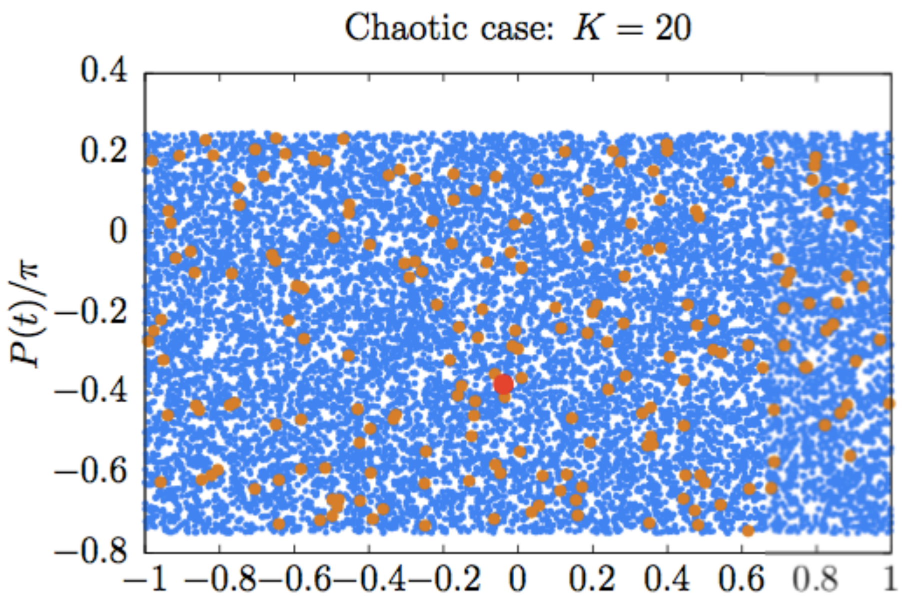
"exponential deviation of trajectories"

$$\left|\frac{\partial x(t)}{\partial x_0}\right| \sim e^{\lambda t}$$ Lyapunov exponent



Poincaré section





MV Berry - AIP Conference proceedings, 1978

Scrambling 1.

Quantum chaos

"hypersensitivity to perturbations of H"

On states:

1984 Peres

$$m(t) = \langle \psi_0 | e^{i(\hat{H}+\delta\hat{B})t}\, e^{-i\hat{H}t} | \psi_0 \rangle$$

Loschmidt echo

2001 Levstein-Jalambert-Pastawski

In operator space:

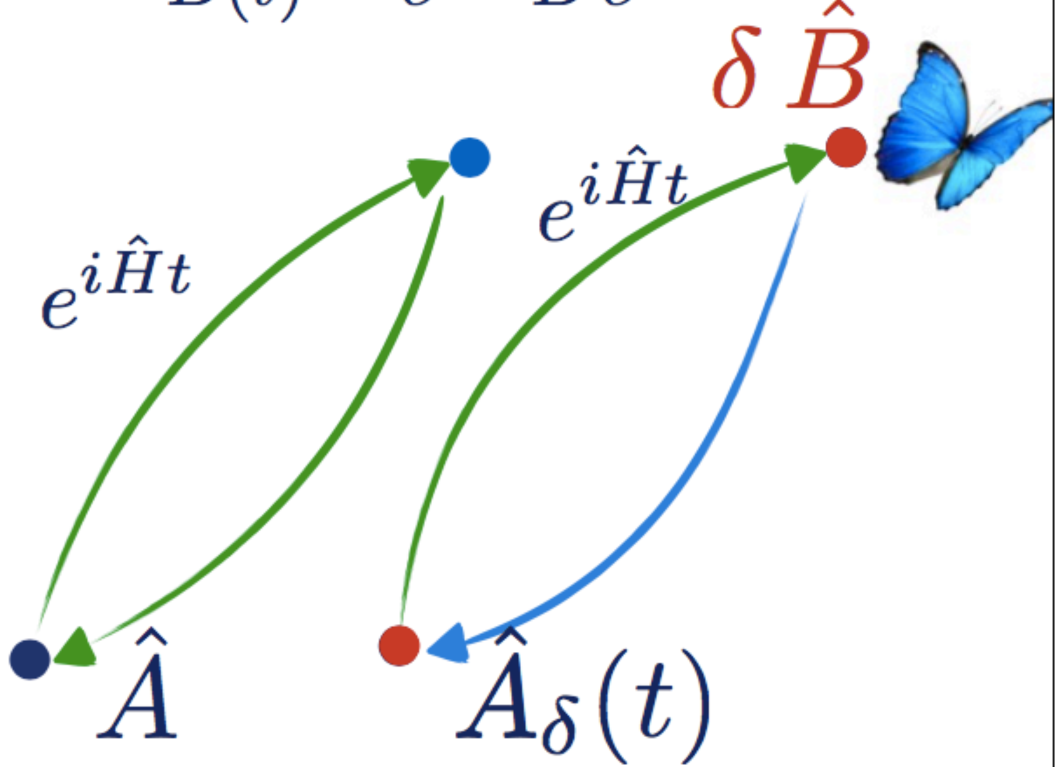
$$\hat{B}(t) = e^{i\hat{H}t}\,\hat{B}\,e^{-i\hat{H}t}$$

$$\hat{A}_\delta(t) = e^{i\delta\hat{B}(t)}\,\hat{A}\,e^{i\delta\hat{B}(t)}$$

$$\simeq \hat{A} + i\delta\,[\hat{B}(t), \hat{A}]$$

$$\langle(\hat{A}_\delta(t) - \hat{A})^2\rangle =$$

$$-\delta^2\,\langle[\hat{B}(t), \hat{A}]^2\rangle$$



The square commutator

Larkin and Ovchinnikov. 1969 SOVIET PHYSICS JETP

QUASICLASSICAL METHOD IN THE THEORY OF SUPERCONDUCTIVITY

$$C(t) = -\langle\, [\hat{x}(t), \hat{p}(0)]^2 \rangle \xrightarrow{\text{canonical quantization}} \hbar^2 \,\{x(t), p_0\}^2 = \hbar^2 \left(\frac{\partial x(t)}{\partial x_0} \right)^2$$

Larkin, Ovchinnikov - Sov Phys JETP, 1969

Kitaev. 2015

- many-body system
- to generic operators
- SYK model

(Majorana fermions: all to all random interaction)

$$C(t) \sim e^{\lambda_Q t}$$

What is scrambling?

$$[\hat{B}, \hat{A}] = 0$$

$$\hat{B}(t) = e^{i\hat{H}t}\hat{B}e^{-i\hat{H}t} = \hat{B} + it[\hat{H}, \hat{B}] - \frac{t^2}{2}[\hat{H}, [\hat{H}, \hat{B}]] + \mathcal{O}(t^3)$$

$$[\hat{B}(t), \hat{A}] \neq 0$$

scrambling: non-commutativity induced by the dynamics!

$$C(t) = -\langle [\hat{B}(t), \hat{A}\,]^2 \rangle_\beta = \langle \hat{B}(t)\,\hat{A}\,\hat{A}\,\hat{B}(t) \rangle + \langle \hat{A}\,\hat{B}(t)\,\hat{B}(t)\,\hat{A}\,\rangle$$
$$-\langle \hat{B}(t)\,\hat{A}\,\hat{B}(t)\,\hat{A}\,\rangle - \langle \hat{A}\,\hat{B}(t)\,\hat{A}\,\hat{B}(t)\,\rangle$$

"out-of-time ordered correlators"
OTOC

expectation for a "chaotic quantum system"

$$\sim \epsilon\, e^{\lambda_Q t} \quad 0 \leq \lambda_Q \leq 2\pi T$$

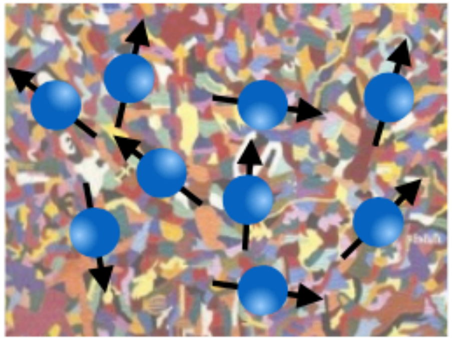
underlying classical limit

Maldacena, Shenker, Stanford - Journal of High Energy Physics, 2016

Scaffidi, Altman - arXiv:1711.04768, 2017
Cotler, Ding, Penington - arXiv:1704.02979, 2017

Non-exponential behavior of $C(t)$

Disordered systems (+ interactions)



- extended (thermal) phase
- MBL phase $C(t) \sim t^{\alpha}$

Chen, Zhou, Huse, Fradkin - Annalen der Physik, 2017

Short range on the lattice

- extensive operators $\hat{A} \equiv \sum_i \hat{\sigma}_i$
- lattice models
- local interactions

$$c(t) \leq A\, t^{3d}$$

Kukuljan, Grozdanov, Prosen - Phys. Rev. B, 2017

!!! relevant in IQ

Hosur, Qi, Roberts, Yoshida - Journal of High Energy Physics, 2016

Scrambling 1. 

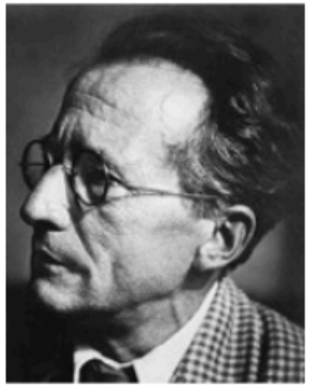
scrambling: non-commutativity of operators induced by the dynamics

square-commutator: introduced in quanto chaos goes exponentially:classical underlying

entanglements spreading 2.

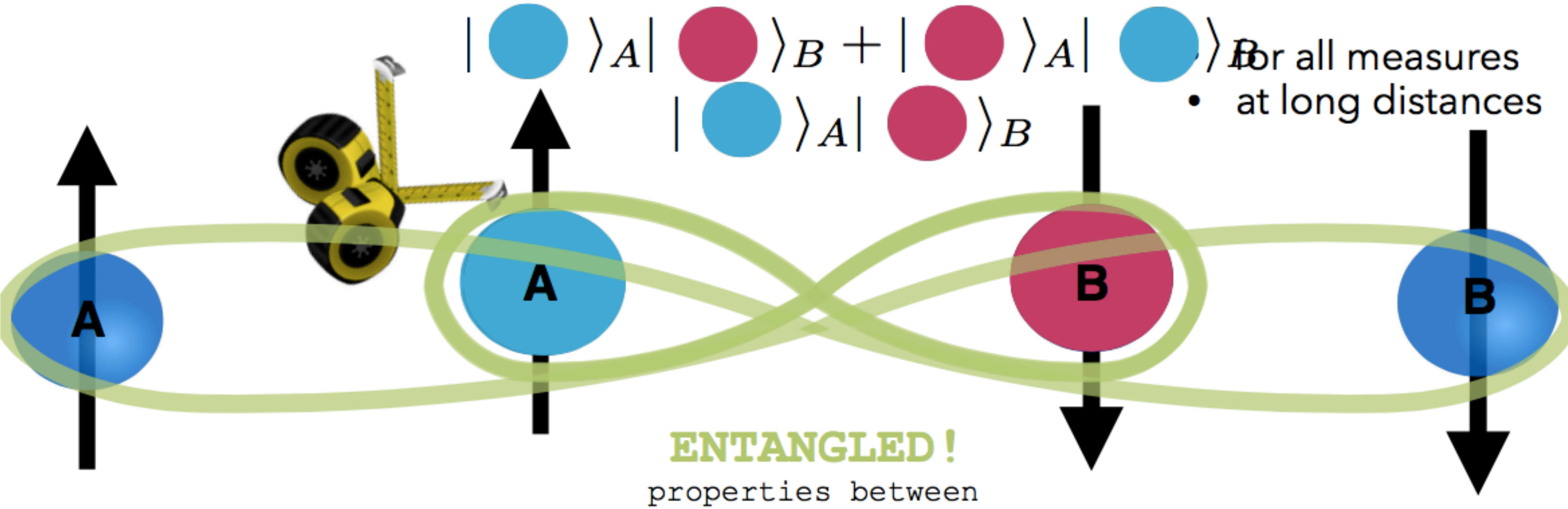
Entanglement

> "*entanglement* is rather **the** characteristic trait of quantum mechanics."



E. Schrödinger, 1935

QUANTUM WORLD: more than an object



- for all measures
- at long distances

ENTANGLED!

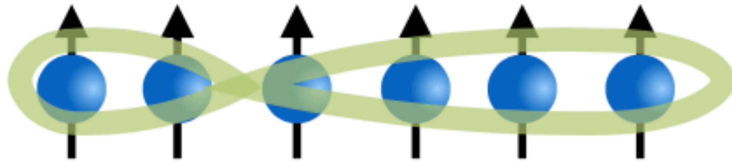
properties between the two systems are correlated

Entanglements dynamics

entanglement entropy

$$S_L(t) = -\mathrm{Tr}(\hat{\rho}_L \log \hat{\rho}_L)$$

Bipartite entanglement

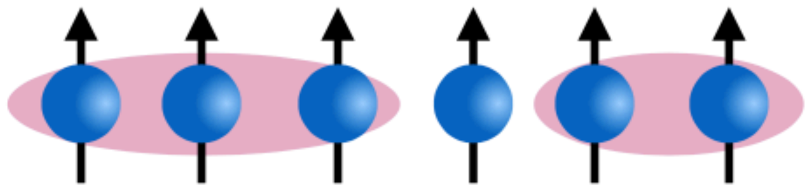


✓

Quantum Fisher Information

$$f_Q(t)$$

Multipartite entanglement



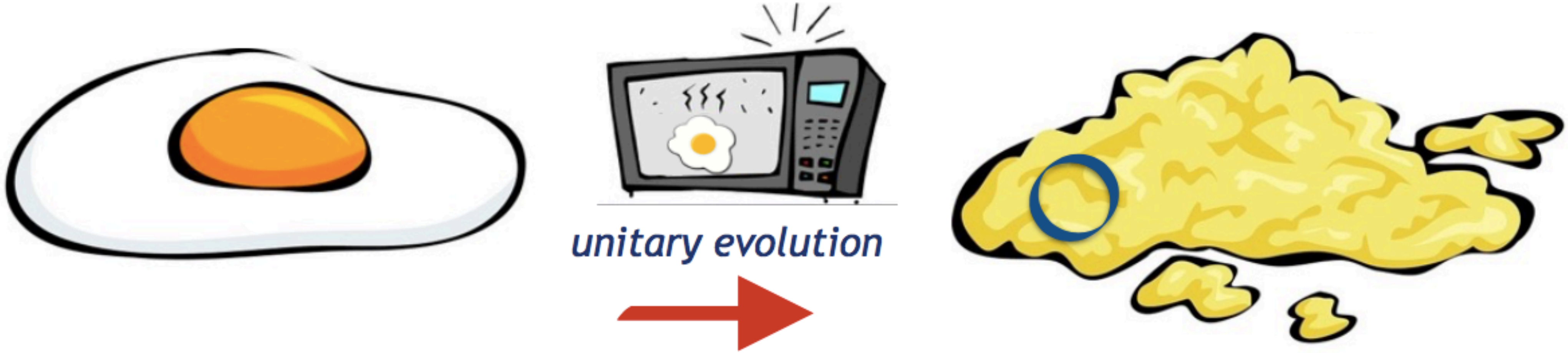
if $f_Q \sim N$
globally entangled state!

bound on the *size of the biggest entangled block*

?!

Scrambling and entanglement

spreading of information ≟ entanglement dynamics



globally ⟶ pure state

locally ⟶ observables thermalize: initial conditions are lost

the information is hidden non-locally in the correlations between subsystems: entanglement

Scrambling 1. 

scrambling: non-commutativity of operators induced by the dynamics

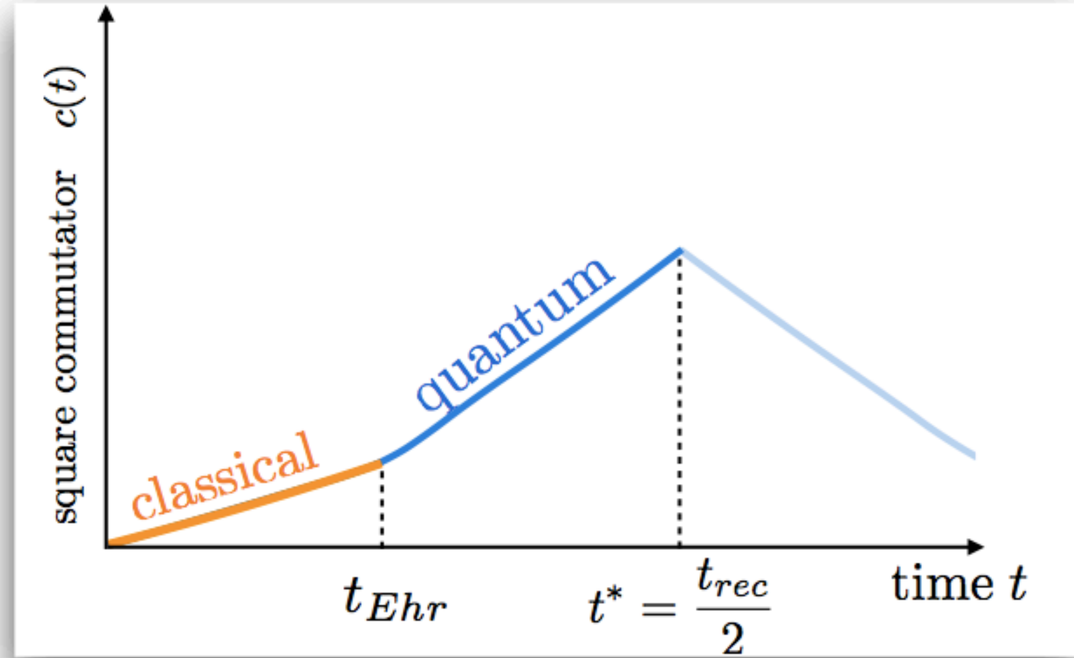
square-commutator: introduced in quanto chaos goes exponentially:classical underlying

entanglements spreading 2. 

globally information is conserved: hidden non-locally in entanglement

3. long range spin chains

life beyond semi-classics



chaotic systems

- exponential
- saturation
- $t_{rec} \sim e^N$

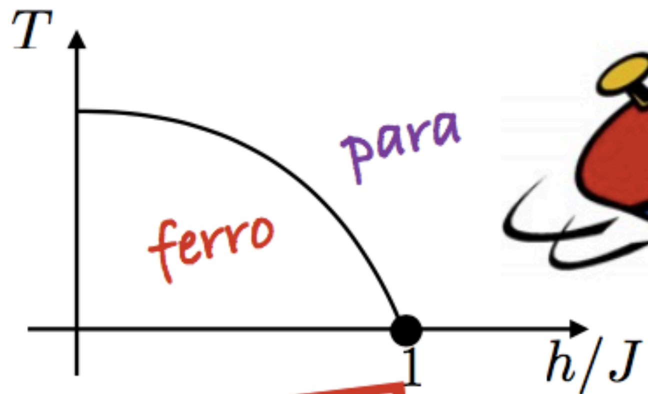
regular systems

- polynomial
- polynomial growth
- $t_{rec} \sim N$
- different from entanglement!

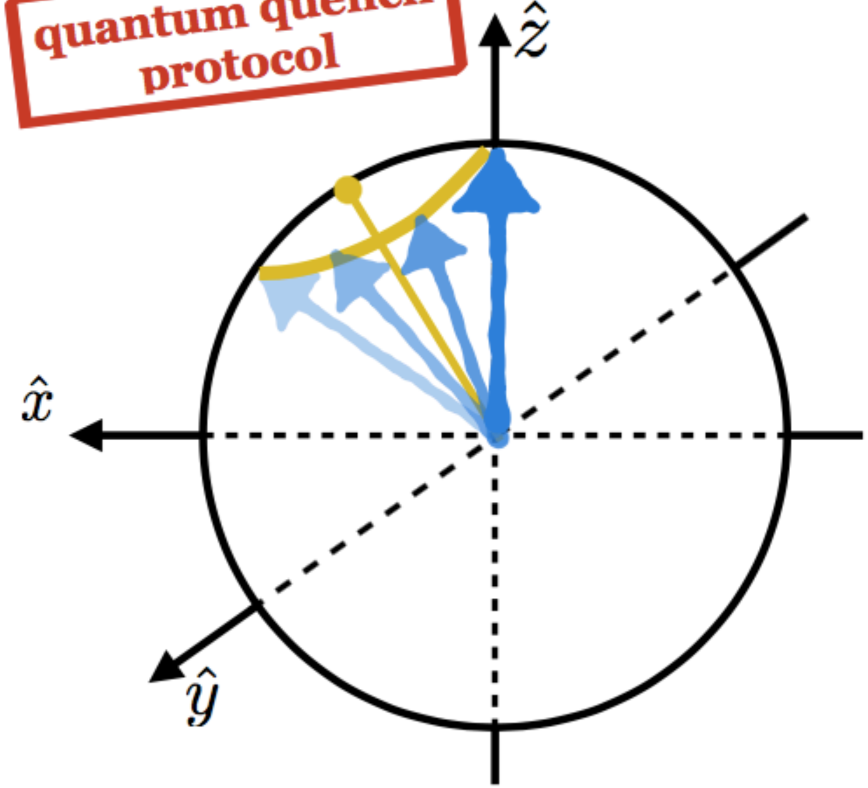
The model. Lipkin-Meshkov-Glick

Infinite range Ising model

$$H(h) = -\frac{2J}{N}\sum_{ij}^{N} S_i^z\, S_j^z - \cancel{2\, h_f \sum_{i}^{N} S_i^x}$$



quantum quench protocol



- solvable
- $[\hat{S}^2, \hat{H}] = 0, \;\; \vec{S} = \sum_i \vec{S}_i$
- semiclassical limit $\hbar_{eff} \sim 1/N$
- initial state $|\psi_0\rangle = |\uparrow\uparrow \ldots \uparrow\rangle$

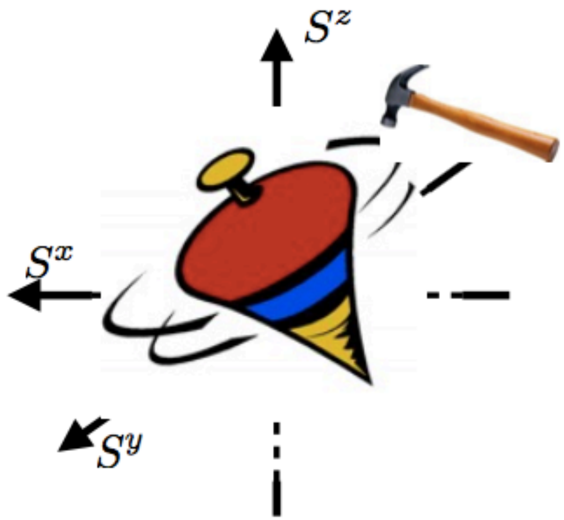
- long range $J_{ij} \sim \dfrac{1}{|i-j|^\alpha}$

Sciolla, Biroli - Phys. Rev. Lett., 2010

Lipkin, Meshkov, Glick - Nuclear Physics, 1965

The kicked top

$$\hat{H} = \hat{H}_{LGM} \quad - \frac{2\,K}{N}\hat{S}_z^{\,2} \sum_{n=-\infty}^{\infty} \delta(t - n\tau)$$



- $[\hat{S}^2, \hat{H}] = 0,\ \ \vec{S} = \sum_i \vec{S}_i$ collective spin

- Floquet theory $\quad \hat{U} = \hat{U}_{\text{kick}} \exp\left[-i\hat{H}_{\text{LGM}}\,\tau\right]$

  with $\hat{U}_{\text{kick}} \equiv \exp\left[-i\frac{2\,K}{N}\,\hat{S}_z^{\,2}\right]$

- semiclassical limit $\hbar_{eff} \sim 1/N$

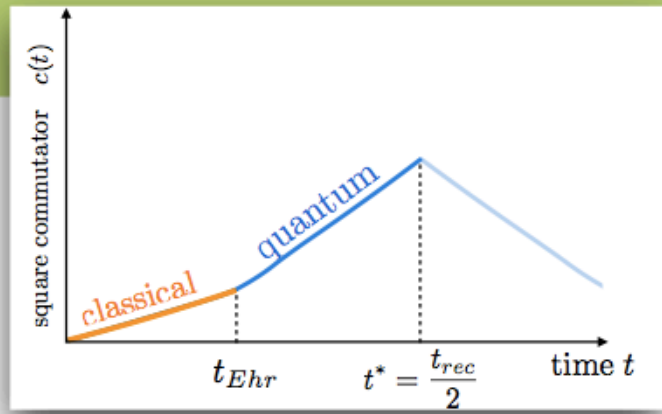
Haake - Springer, 2013

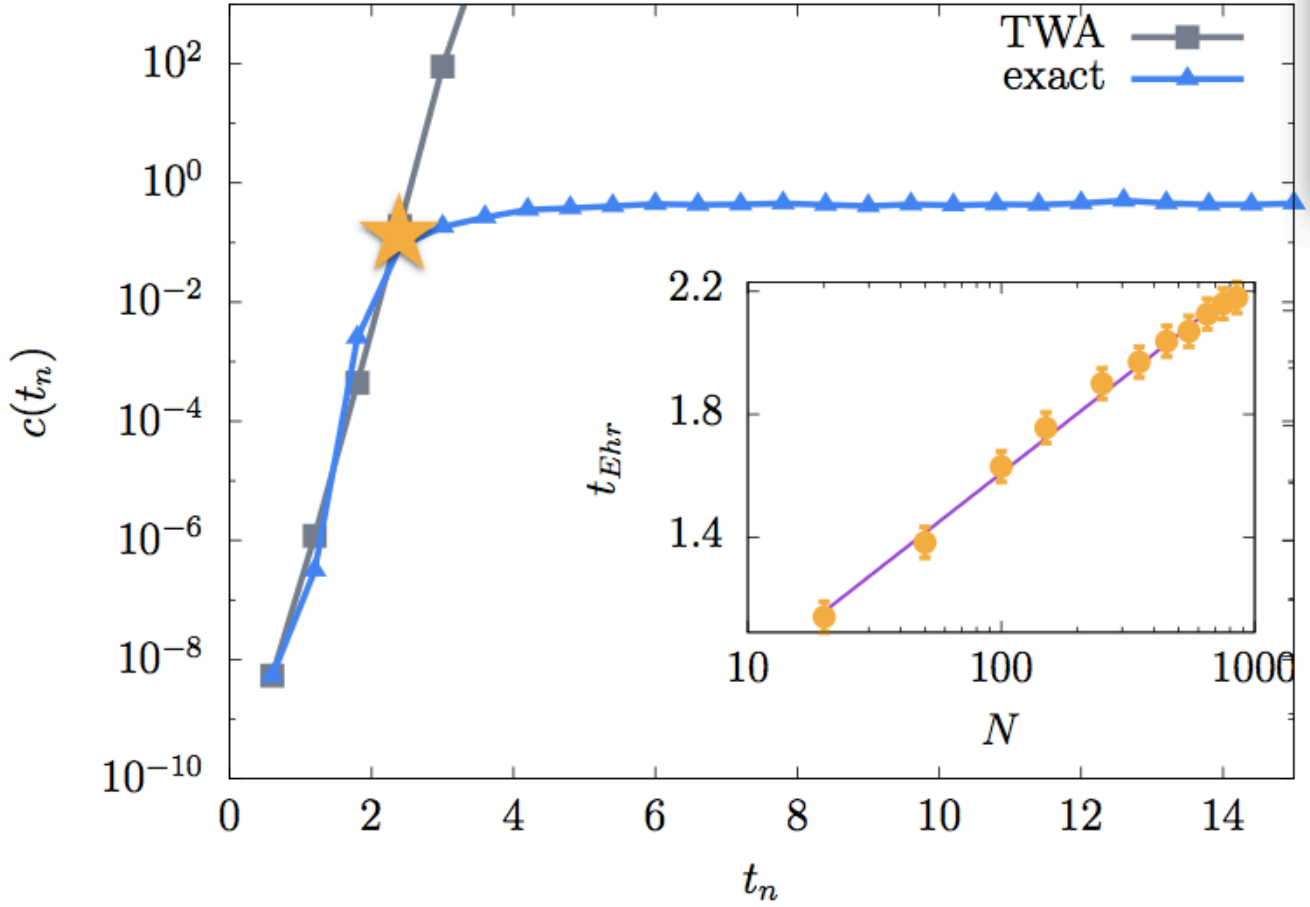
Scrambling in the chaotic top

$K = 20$

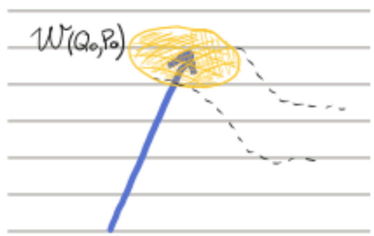
$$c(t) = -\langle [\hat{m}^z(t), \hat{m}^z]^2 \rangle$$

1. in chaotic systems
   quantum chaos = classical chaos
   for $t \leq t_{Ehe}$





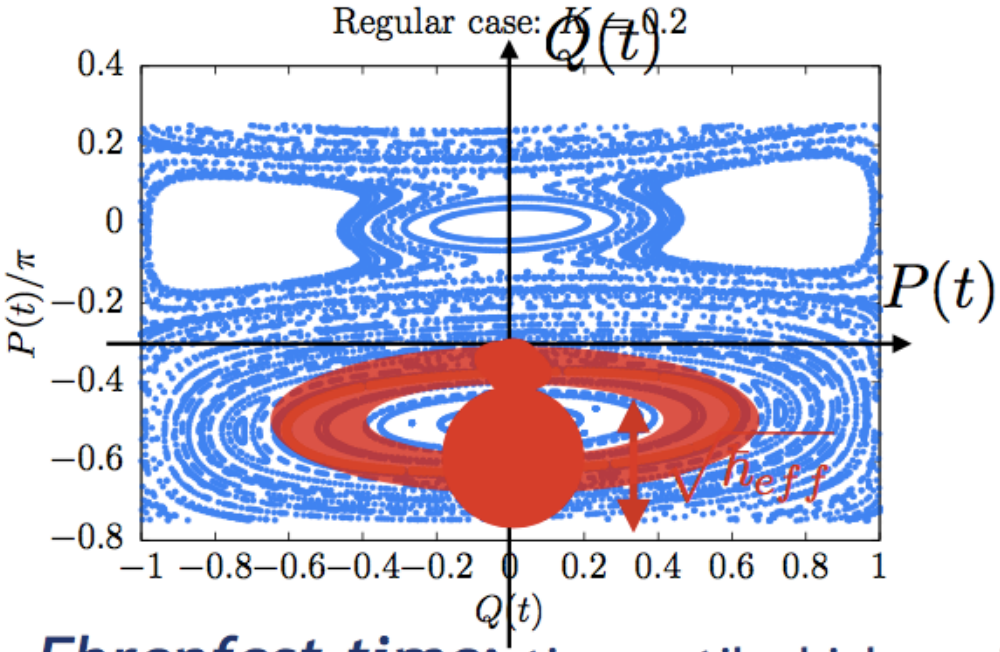
Truncated Wigner Approximation

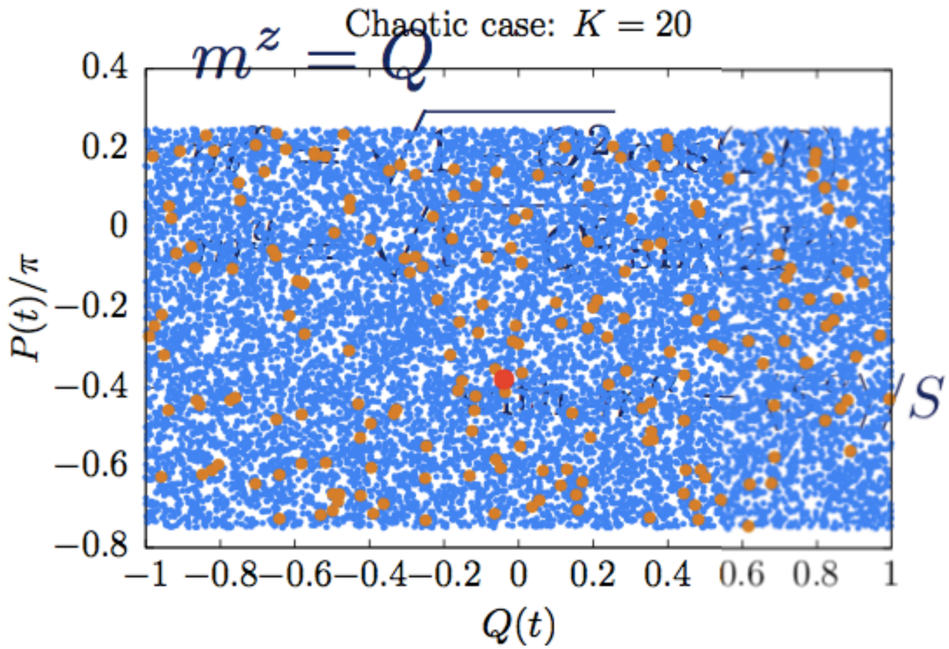


Classical and quantum chaos

$$\hat{U} = \exp\left[-i\frac{2\,K}{N}\,\hat{S}_z^2\right]\,e^{-i\hat{H}_{\rm LGM}\,\tau}$$

classical limit $\hbar_{eff} = \hbar/N$





Ehrenfest time: time until which semi-classics holds

Regular	Chaotic
$\sqrt{\hbar_{eff}}\,t \sim 1$ $\quad t_{\rm Ehr} \sim \sqrt{N}$	$\sqrt{\hbar_{eff}}e^{\lambda t} \sim 1$ $\quad t_{Ehr} \sim \log N$

Recurrence time: time at which the wave-packet regenerates

Regular	Chaotic
$t_{\rm rec} \sim N$	$t_{rec} \sim e^N$

---

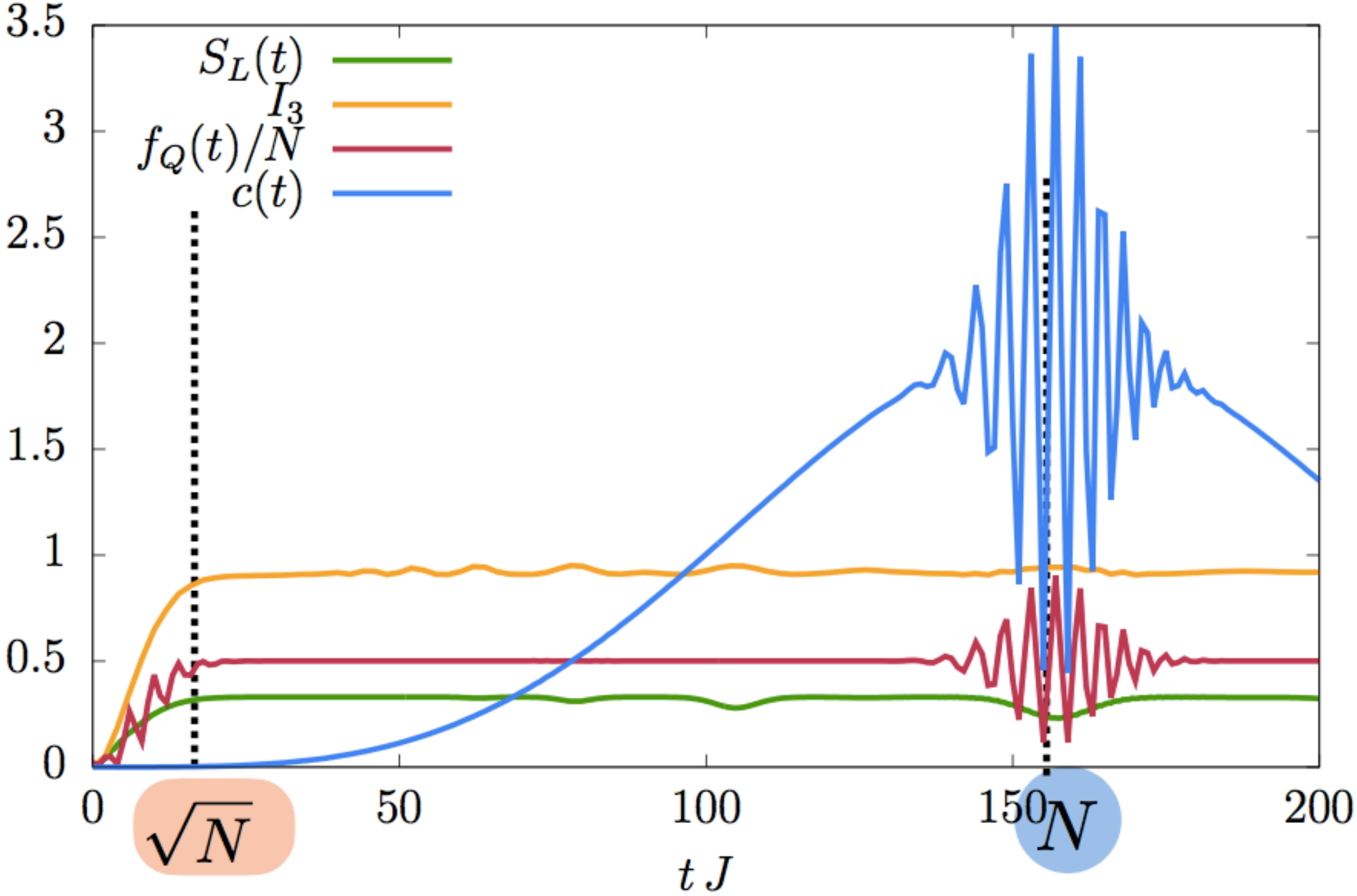
quantum spectral properties of Floquet spectrum: transition from Poisson to Wigner-Dyson distribution

Information dynamics in the LGM $K = 0$

$$c(t) = -\langle\, [\hat{m}^z(t), \hat{m}^z]^2 \,\rangle$$

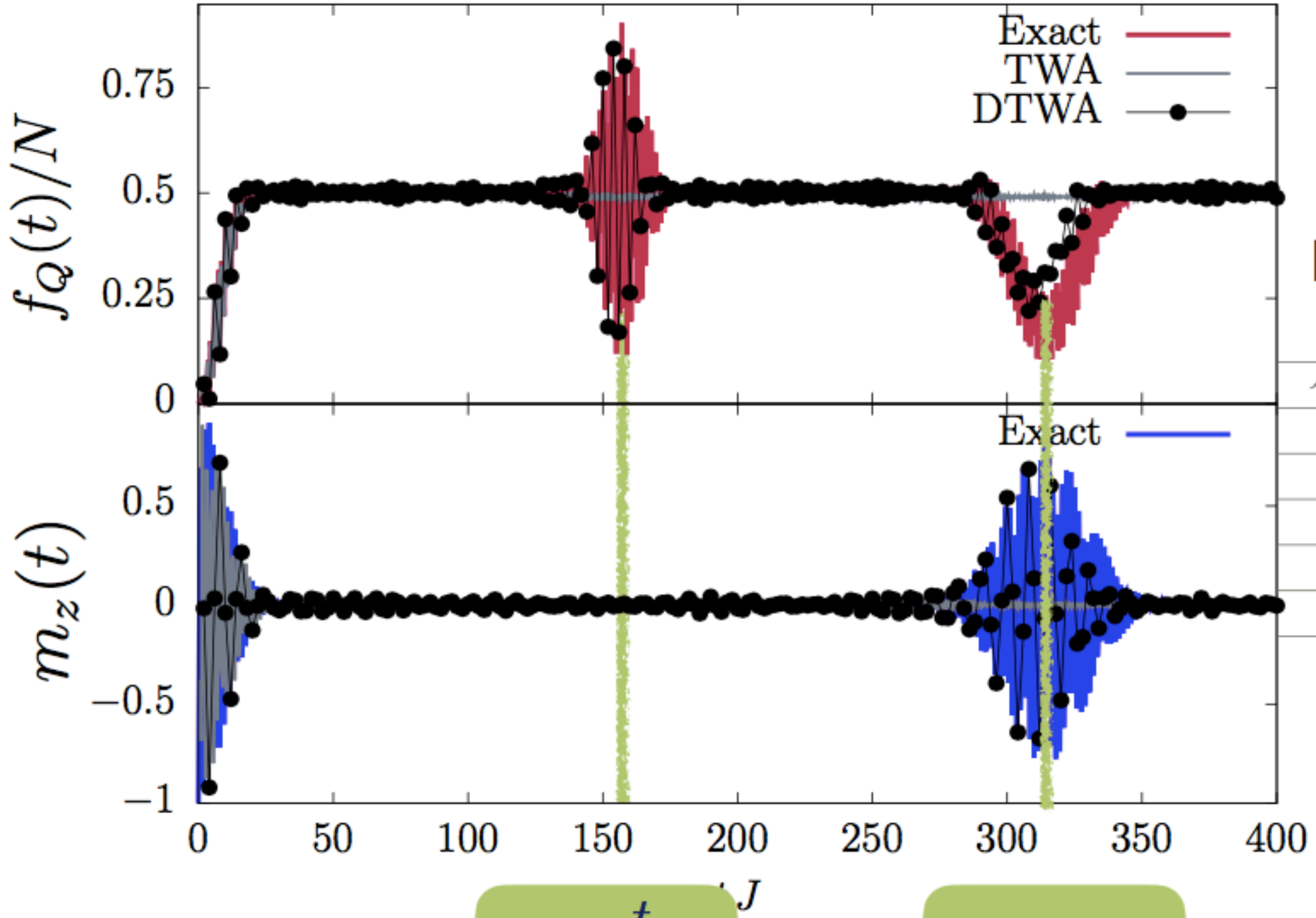
$$S_L = \mathrm{Tr}\,(\hat{\rho}_L \log \hat{\rho}_L\,)$$

2. scrambling goes beyond entanglement

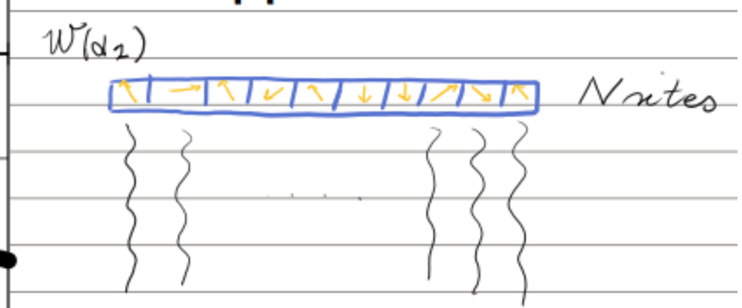


Entanglement and semi-classics

3.entanglement is a state dependent property



Discrete Truncated Wigner Approximation



$$t^* = \frac{t_{rec}}{2}$$

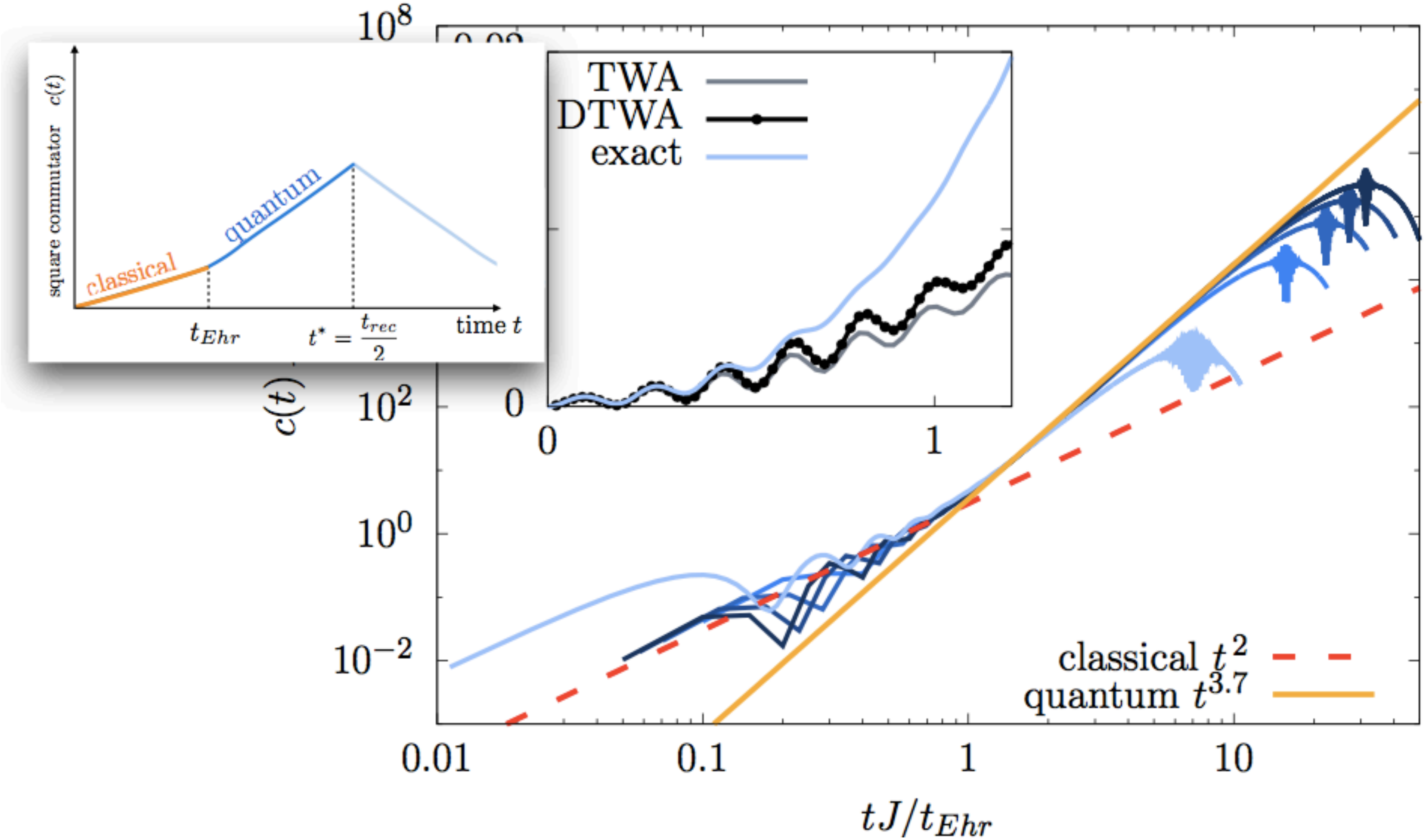
$$t_{rec} = N$$

Scrambling beyond semiclassics

$c(t)$

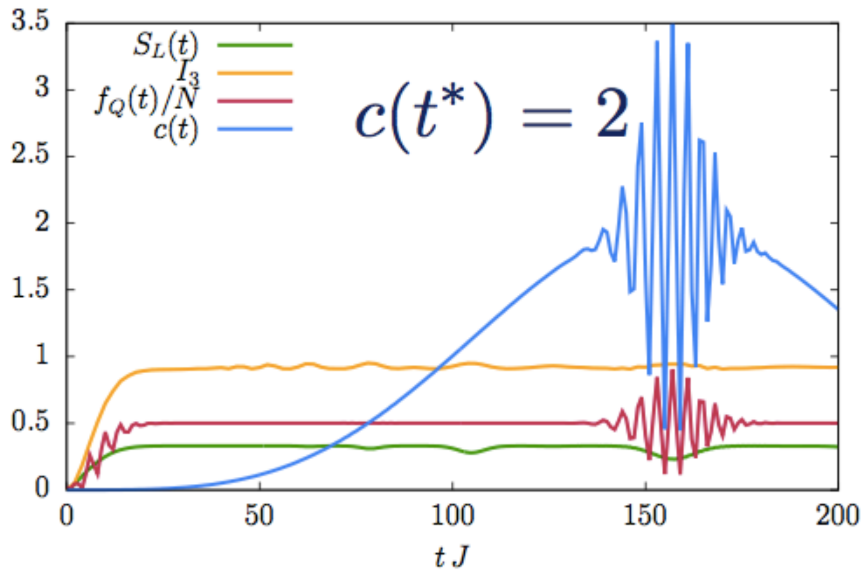
- $t \leq t_{Ehr}$ *semi-classical regime* $\sim \dfrac{t^2}{N^3}$
- $t_{Ehr} \leq t \leq t^*$ *quantum regime* $c(t^*) = 2$

$$t^* = \frac{t_{rec}}{2}$$



Quantum regime and the operator's growth

$$c(t) = -\frac{1}{N^4}\left\langle \left[\hat{S}^z(t), \hat{S}^z\right]^2 \right\rangle$$



4. related to the operator's support growth: non-perturbative

$$\hat{S}_z(t) = \sum_{k=0}^{\infty} \frac{(-it)^k}{k!}[\hat{H},[\ldots,[\hat{H},\hat{S}_z]]]$$

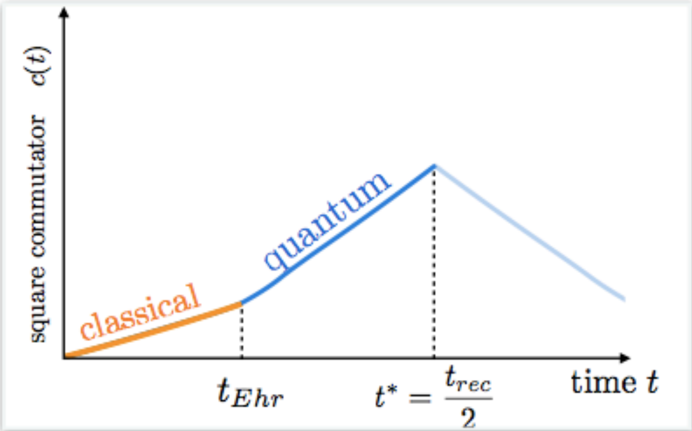
string of length N

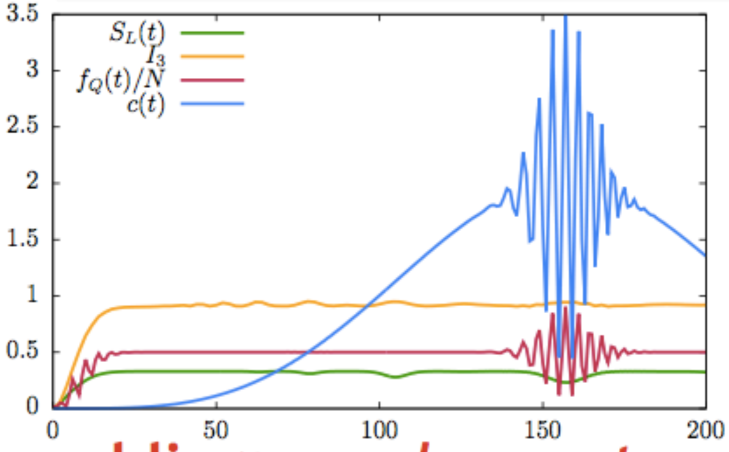
$$= \sum_{\alpha_1\in\{x,y,z\}} a^{\alpha_1}(t)\,\widehat{S}^{\alpha_1} + \sum_{\alpha_1,\alpha_2} b^{\alpha_1\alpha_2}(t)\,\widehat{S}^{\alpha_1}\widehat{S}^{\alpha_2} + \cdots + \sum_{\alpha_1,\cdots,\alpha_N} z^{\alpha_1\cdots\alpha_N}(t)\,\widehat{S}^{\alpha_1}\widehat{S}^{\alpha_2}\cdots\widehat{S}^{\alpha_N}$$

- scrambling always symmetric around $t^* = \frac{t_{rec}}{2}$

Thanks!

Conclusions





scrambling: ***purely quantum, not entanglement***

1. in chaotic systems
   quantum chaos = classical chaos
   $t \leq t_{Ehe}$
2. scrambling goes beyond
3. entanglement is a state dependent property 
4. related to the operator's support growth: non-perturbative

Perspectives

- breaking of integrability
- use operator space entropy

Wigner representation and TWA

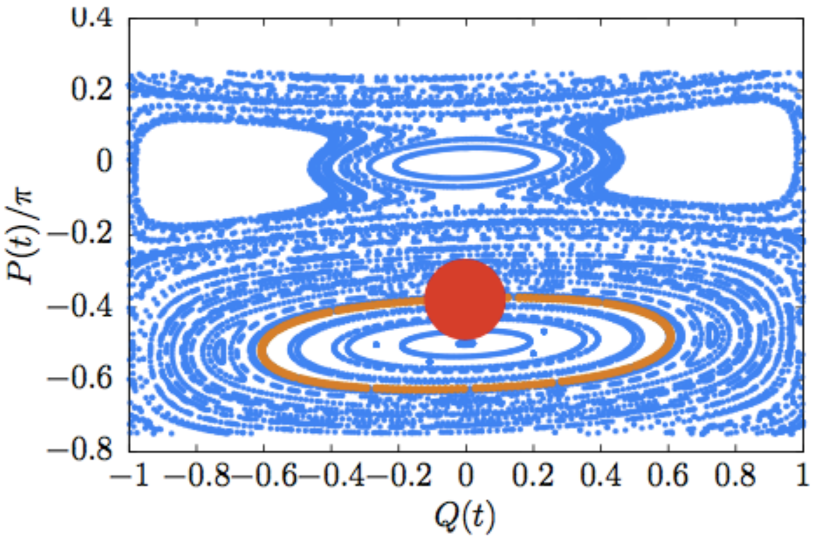
Hilbert space ⟷ **Continous Phase Space**

- density matrix $\hat{\rho}$ — Wigner function $W(q,p)$
- operators $\hat{O}$ — Weyl transform $O_w(p,q)$

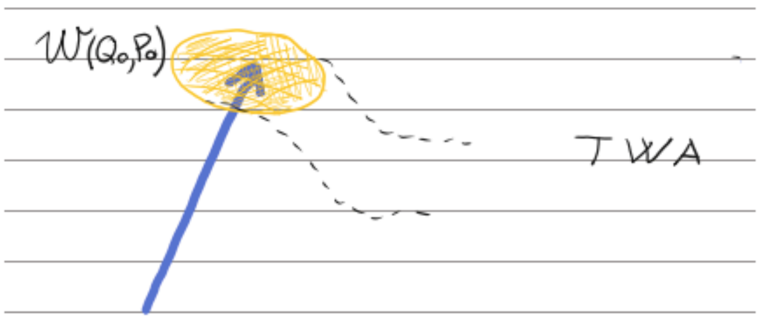
... ...

- expectation values — **Truncated Wigner Approximation**

$$\langle \hat{O}(t) \rangle = Tr[\,\hat{\rho}_0 \hat{O}(t)\,] \simeq \int dq_0\, dp_0\, W(q_0,p_0)\, O_w(q^{cl}(t), p^{cl}(t))$$

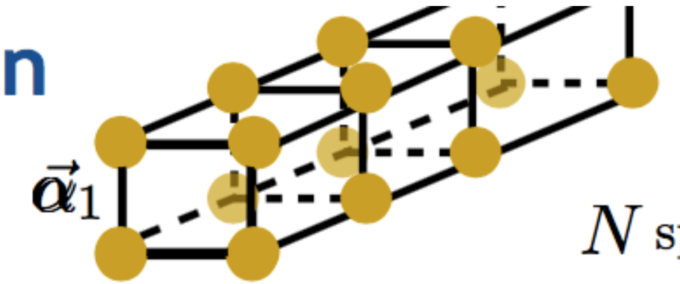


Montecarlo Sampling



classical evolution + average over the initial Wigner distribution

Discrete Wigner representation and DTWA



N spin $\frac{1}{2}$

Hilbert space ⟷ Discrete Phase Space

- density matrix $\hat{\rho}$
- operators $\hat{O}$

...

Wigner function $W(\vec{\alpha})$

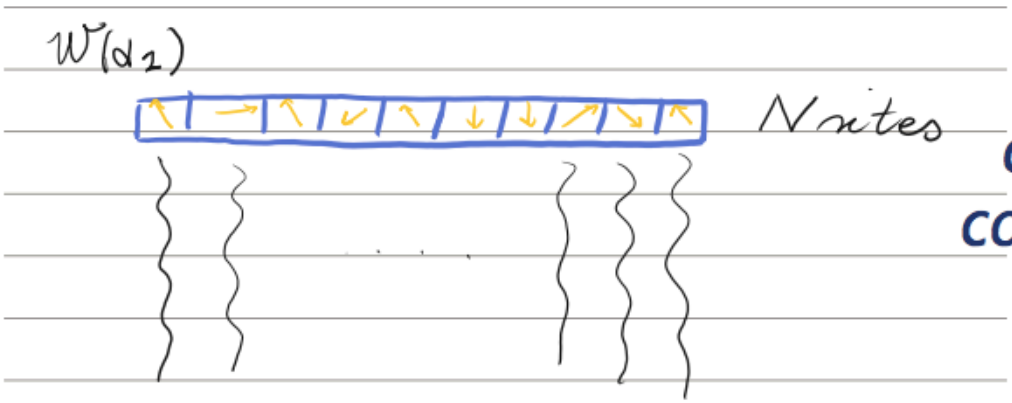
Weyl transform $O_W(\vec{\alpha})$

...

- expectation values

$$\langle \hat{O}(t) \rangle = Tr[\,\hat{\rho}_0 \hat{O}(t)\,] = \sum_{\vec{\alpha}} W(\vec{\alpha}_0) O_W(\vec{\alpha}(t))$$

Montecarlo Sampling



discretization of the initial condition + classical evolution $3N$

Schachenmayer, Pikovski, Rey - Phys. Rev. X, 2015

Wootters - Annals of Physics, 198?

Entanglement structure

Numerics with MPS-TDVP

matrix product state time-dependent variational principle

$$H(h) = -\frac{2J}{N(\alpha)} \sum_{i \neq j = 1}^{N} \frac{S_i^z S_j^z}{|i-j|^\alpha} - 2h \sum_{i=1}^{N} S_i^x$$

$0 \leq \alpha < 1$	$1 < \alpha < 2$	$\alpha > 2$
$f_Q(\infty) \sim N$	$f_Q(t) \sim \text{const}$	$f_Q(t) \sim \text{const}$
$S_L(t) \sim \log t$	$S_L(t) \sim t^\beta$ with $\beta < 1$	$S_L(t) \sim t$
$S_L(\infty) \sim \log L$		$S_L(\infty) \sim L$
...	...	...

General structure!
induced by entanglement's
monogamy

Haegeman, Lubich, Oseledets, Vandereycken... - Phys. Rev. B, 2016