

# Gravitational decoherence: a general non relativistic model

L. Asprea,<sup>1,2,\*</sup> G. Gasbarri,<sup>3,2,†</sup> and A. Bassi<sup>1,2,‡</sup>

<sup>1</sup>*Department of Physics, University of Trieste, Strada Costiera 11, 34151 Trieste, Italy*

<sup>2</sup>*Istituto Nazionale di Fisica Nucleare, Trieste Section, Via Valerio 2, 34127 Trieste, Italy*

<sup>3</sup>*Department of Physics and Astronomy, University of Southampton, Highfield Campus, SO17 1BJ, United Kingdom*

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We derive a general quantum master equation for the dynamics of a scalar bosonic particle interacting with a weak, stochastic and classical external gravitational field. The dynamics predicts decoherence in position, momentum and energy. We show how our master equation reproduces the results present in the literature by taking appropriate limits, thus explaining the apparent contradiction in their dynamical description. Our result is relevant in light of the increasing interest in the low energy quantum-gravity regime.

## I. INTRODUCTION

One of the greatest predictions of general relativity is the existence of gravitational waves, which can be thought of as small perturbations of the metric propagating through spacetime at the speed of light [1–4]. They are of fundamental interest in many branches of physics, such as cosmology, theoretical physics and astrophysics, and their recent first detection [5–9] has opened thrilling new horizons for research and a huge effort is being put into the construction of ever more sophisticated detectors [10].

Most gravitational waves that arrive on the Earth are produced by different unresolved mechanisms and sources [11, 12], and thus result in a stochastic perturbation of the flat spacetime background. Within the framework of quantum theory, this stochastic background affects the dynamics of matter propagation [13, 14] and, when the quantum state is in a superposition, it leads to decoherence effects, as typical of noisy environments. Since quantum superpositions are very sensitive to small variations of the surrounding en-

vironment, quantum interferometers have the potential to detect a stochastic gravitational background [15–17]. Different models for the description of this phenomenon have been proposed [18–24]. However, they do not agree on the decoherence mechanism (the preferred basis and rates) at which it takes place. With this work we clarify this issue. We derive a general non relativistic model of gravitational decoherence starting from the dynamics of a scalar bosonic field coupled to a weak gravitational perturbation. We show how this model recovers the results present in the literature as appropriate limiting cases.

The paper is organized as follows. In section II we derive the equations of motion in Hamiltonian form for a scalar bosonic field minimally coupled to a weakly perturbed flat metric. We then specialize such equation to the non relativistic regime in section III and proceed with the canonical quantization of the bosonic field in the single particle sector, obtaining a Schrödinger like equation for a test particle interacting with a weakly perturbed gravitational field.

In section IV we specialize to the case of a stochastic gravitational perturbation and derive the corresponding master equation. We discuss the decoherence effect in sections V and VI with explicit reference to the preferred

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\*Electronic address: lorenzo.asprea@phd.units.it

†Electronic address: g.gasbarri@soton.ac.uk

‡Electronic address: abassi@ts.infn.it

eigenbasis and characteristic decoherence time. In the same sections we show under which assumptions our master equation is able to reproduce the apparently contradictory results of [19–21] and [22], thus solving the preferred basis puzzle.

## II. HAMILTONIAN EQUATIONS OF MOTION

We first derive the equations of motion (EOM) for a scalar bosonic field minimally coupled to linearized gravity. We start from the action for the charged Klein Gordon field in curved spacetime [25]:

$$S = \int d^4x \sqrt{-g} \mathcal{L} \quad (1)$$

with the Lagrangian density:

$$\mathcal{L} = (c^2 g^{\mu\nu} \nabla_\mu \psi^* \nabla_\nu \psi - \frac{m^2 c^4}{\hbar^2} \psi^* \psi) \quad (2)$$

where  $\nabla_\mu$  is the covariant derivative with respect to the Christoffel connection. We write the metric as the sum of a flat background  $\eta_{\mu\nu} = \text{diag}(+ - - -)$ , and a perturbation  $h_{\mu\nu}$ :

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (3)$$

We are interested in studying the dynamics of the Klein Gordon field in presence of a weak gravitational perturbation. Therefore we perform a Taylor expansion of the action around the flat background metric and truncate the series at the first perturbative order. Thus, we obtain the effective Lagrangian  $\mathcal{L}_{eff}$  acting on flat spacetime:

$$\begin{aligned} S &= \int d^4x \left[ c^2 (\eta^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - \frac{m^2 c^2}{\hbar^2} \psi^* \psi) \left( 1 + \frac{\text{tr}(h^{\mu\nu})}{2} \right) + \right. \\ &\quad \left. - c^2 h^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi + O(h^2) \right] \\ &=: \int d^4x (\mathcal{L}_{eff} + O(h^2)) \end{aligned} \quad (4)$$

Note that in doing so we are implicitly restricting the analysis to the class of reference frames in which the coordinates are described by rigid rulers, which are negligibly

affected by the gravitational perturbation. This assumption though reasonable, as measuring devices are held together by intra and inter molecular forces, is arbitrary (it may be possible that a gravitational perturbation bends a measuring device).

The equations of motion for the matter field are obtained (at first order in the perturbation  $h_{\mu\nu}$ ) from the Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}_{eff}}{\partial \psi^*} - \partial_\alpha \frac{\partial \mathcal{L}_{eff}}{\partial \partial_\alpha \psi^*} = 0 \quad (5)$$

and in the harmonic gauge [33] they read:

$$\begin{aligned} &\left[ -\partial_t^2 + c^2 (1 + h^{00}) \nabla^2 + 2ch^{0i} \partial_t \partial_i + c^2 h^{ij} \partial_i \partial_j + \right. \\ &\quad \left. - \frac{m^2 c^4}{\hbar^2} (1 + h^{00}) + O(h^2) \right] \psi = 0 \end{aligned} \quad (6)$$

We are interested in the description of the dynamics of a positive energy particle system in the non relativistic limit. In such a limit, the particle and antiparticle sectors are non interacting with one another, that is to say, the EOM (6) can be recast to a system of two uncoupled equations, one for each species sector. While this is evident and straightforward for the free case, for an interacting theory the decoupling is very complicated and achievable only perturbatively.

The first step is to explicitly express the field in a two component form. This can be done following the Feshbach-Villars formulation [26]. Accordingly we define a new field:

$$\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix} \quad (7)$$

such that:

$$\begin{cases} \psi = \phi + \chi \\ i\hbar (\partial_t - ch^{0i} \partial_i) \psi = mc^2 (\phi - \chi) \end{cases} \quad (8)$$

We note that such a formulation does not allow for a probabilistic interpretation of the field  $\Psi$ , as the conserved charged  $Q$  associated to the internal  $U(1)$  symmetry ( $\psi \rightarrow e^{ie}\psi$ ;  $\psi^* \rightarrow e^{-ie}\psi^*$ ) via Noether's Theorem

reads:

$$Q = 2e mc^2 \int d^3x \begin{pmatrix} \phi \\ \chi \end{pmatrix} \sigma_3 \left(1 + \frac{\text{tr}(h^{\mu\nu})}{2} - h^{00}\right) \quad (9)$$

instead of the required:

$$\rho = 2e mc^2 \int d^3x \begin{pmatrix} \phi \\ \chi \end{pmatrix} \sigma_3 \begin{pmatrix} \phi \\ \chi \end{pmatrix} \quad (10)$$

We therefore apply the transformation:

$$\begin{cases} T &= (1 + \frac{\text{tr}(h)}{4} - \frac{h_{00}}{2}) \\ \Psi &\rightarrow T\Psi \end{cases} \quad (11)$$

so that, in the new representation, the squared modulus of the field can be regarded as a probability density in the non relativistic limit.

With the help of Eq. (8) and after some algebra (See Appendix A) the EOM (6) read:

$$i\hbar\partial_t\Psi = [mc^2\sigma^3 + \mathfrak{E} + \mathcal{O}]\Psi \quad (12)$$

where:

$$\begin{aligned} \mathfrak{E} &= \frac{mc^2}{2}h^{00}\sigma_3 - \frac{\hbar^2}{2m}(1+h^{00})\sigma_3\nabla^2 - \frac{\hbar^2}{2mc}\partial_t(h^{0i})\sigma_3\partial_i \\ &\quad - \frac{\hbar^2}{2m}h^{ij}\sigma_3\partial_i\partial_j + i\hbar ch^{0i}\partial_i - \frac{i\hbar}{2}\partial_t\left(\frac{\text{tr}(h^{\mu\nu})}{2} - h^{00}\right) \\ &\quad - \left[\frac{\hbar^2}{4m}\nabla^2(h^{00}) - \frac{i\hbar^2}{8m}\nabla^2(\text{tr}(h^{\mu\nu}))\right]\sigma_3 \end{aligned} \quad (13)$$

$$\begin{aligned} \mathcal{O} &= \frac{imc^2}{2}h^{00}\sigma_2 - \frac{i\hbar^2}{2m}(1+h^{00})\sigma_2\nabla^2 - \frac{i\hbar^2}{2mc}\partial_t(h^{0i})\sigma_2\partial_i \\ &\quad - \frac{i\hbar^2}{2m}h^{ij}\sigma_2\partial_i\partial_j - \left[\frac{i\hbar^2}{4m}\nabla^2(h^{00}) - \frac{i\hbar^2}{8m}\nabla^2(\text{tr}(h^{\mu\nu}))\right]\sigma_2 \end{aligned} \quad (14)$$

are respectively the diagonal and antidiagonal parts of the Hamiltonian  $K = mc^2\sigma^3 + \mathfrak{E} + \mathcal{O}$ , and  $\sigma_i$ ,  $i = 1, 2, 3$  are the Pauli matrices.

In the next section we will decouple the EOM to then take the non relativistic limit.

### III. NON RELATIVISTIC LIMIT AND CANONICAL QUANTIZATION

We want to find a representation of the two component field  $\Psi$  in which the EOM (12) are diagonal. This representation can be found in non relativistic limit following the Foldy-Wouthuysen Method [27], which allows one to write perturbatively (at any order in  $\frac{v}{c}$ ) two decoupled equations, one for each component of the field. The method is operatively characterized by the application of an appropriate transformation  $U$ :

$$\Psi \rightarrow \Psi' = U\Psi \quad (15)$$

$$\begin{aligned} K \rightarrow K' &= U(K - i\hbar\partial_t)U^{-1} \\ &= mc^2\sigma_3 + \mathfrak{E}' + \mathcal{O}' + O(\hbar^2) \end{aligned} \quad (16)$$

such that, in the new representation, the antidiagonal part  $\mathcal{O}'$  is of higher order in  $\frac{v}{c}$  than the diagonal  $\mathfrak{E}'$ . By neglecting  $\mathcal{O}'$  one recovers two decoupled equations. By performing iteratively the transformation, one can always find a representation of the two component field for which the EOM are diagonal at any desired order in  $\frac{v}{c}$ .

In our case, we have that the task is easily achieved by applying the subsequent transformations:

$$\begin{cases} U &= e^{-i\sigma_3\mathcal{O}/(2mc^2)} \\ U' &= e^{-i\sigma_3\mathcal{O}'/(2mc^2)} \\ U'' &= e^{-i\sigma_3\mathcal{O}''/(2mc^2)} \end{cases} \quad (17)$$

after which, with some algebra (see Appendix B), the EOM read:

$$\begin{aligned} i\hbar\partial_t\Psi &= H\Psi \\ &= \left[ mc^2\left(1 + \frac{h^{00}}{2}\right)\sigma_3 - \frac{\hbar^2}{2m}\left(1 + \frac{h^{00}}{2}\right)\nabla^2\sigma_3 + \right. \\ &\quad \left. - \frac{\hbar^2}{2m}h^{ij}\partial_i\partial_j\sigma_3 + i\hbar ch^{0i}\partial_i + \frac{i\hbar}{2}\partial_t(h^{00}) \right. \\ &\quad \left. - \frac{i\hbar}{4}\partial_t(\text{tr}(h^{\mu\nu})) + \frac{\hbar^2}{8m}\nabla^2(\text{tr}(h^{\mu\nu}))\sigma_3 \right]\Psi \\ &\quad + O(c^{-4}) + O(\hbar_{\mu\nu}^2) \end{aligned} \quad (18)$$

Note that as the transformations (17) are generalized unitary [28], they preserve the conserved charge in (9),

i.e. the probability density in the non relativistic limit.

In the non relativistic limit the EOM (18) do not mix the two components  $\phi$  and  $\xi$  of the field (up to a very small correction). As we are interested in the dynamics of particles only, we restrict the analysis to the first field component  $\phi$ .

Since the dynamics preserves the probability density, we are allowed to apply the canonical quantization prescription and impose the equal time commutation relations:

$$\begin{aligned} [\hat{\phi}(t, \mathbf{x}), \hat{\phi}(t, \mathbf{x}')] &= [\hat{\phi}^\dagger(t, \mathbf{x}), \hat{\phi}^\dagger(t, \mathbf{x}')] = 0 \\ [\hat{\phi}(t, \mathbf{x}), \hat{\phi}^\dagger(t, \mathbf{x}')] &= \delta^3(\mathbf{x} - \mathbf{x}') \end{aligned} \quad (19)$$

to obtain the EOM for the quantum field. The equation thus obtained does not allow for the creation or annihilation of particles. We can thus safely project it onto a single particle sector to obtain the single particle Schrödinger equation:

$$i\hbar\partial_t|\phi(t)\rangle = (\hat{H}_0 + \hat{H}_p)|\phi(t)\rangle \quad (20)$$

with:

$$\begin{aligned} \hat{H}_0 &= mc^2 + \frac{\hat{\mathbf{p}}^2}{2m} \\ \hat{H}_p &= \frac{mc^2}{2}h^{00}(t, \hat{\mathbf{x}}) - \frac{\hbar^2}{4m}h^{00}(t, \hat{\mathbf{x}})\hat{\mathbf{p}}^2 + ch^{0i}\hat{p}_i + \\ &\quad - \frac{1}{2m}h^{ij}(t, \hat{\mathbf{x}})\hat{p}_i\hat{p}_j + \frac{\hbar^2}{8m}\nabla^2(\text{tr}[h^{\mu\nu}(t, \hat{\mathbf{x}})]) + \\ &\quad + \frac{i\hbar}{2}\partial_t(h^{00}(t, \hat{\mathbf{x}})) - \frac{i\hbar}{4}\partial_t(\text{tr}[h^{\mu\nu}(t, \hat{\mathbf{x}})]) \end{aligned} \quad (21)$$

where  $\hat{\mathbf{x}}, \hat{\mathbf{p}}$  are respectively the single particle position and the momentum operator. The term  $H_0$  is the standard free hamiltonian plus an irrelevant global phase  $mc^2$  that can be reabsorbed with the transformation:

$$|\phi(t)\rangle \rightarrow e^{imc^2t/\hbar}|\phi(t)\rangle \quad (22)$$

The term  $\hat{H}_p$  is a perturbation that encodes the interaction between the scalar bosonic particle and a weak, otherwise generic, gravitational perturbation. We note that Eq. (20) correctly reduces to the usual Schrödinger equation for a particle in an external static newtonian

potential:

$$\begin{aligned} i\hbar\partial_t|\phi(t)\rangle &= \left(\frac{\hat{\mathbf{p}}^2}{2m} - m\Phi\right)|\phi(t)\rangle \\ \Phi &= -\frac{c^2h^{00}}{2} \end{aligned} \quad (23)$$

if we consider the external gravitational field to be of the same form of that of the Earth.

The generalization of Eq. (20) to an extended body is not an easy task, as one needs to take into account the degrees of freedom of all the elementary particles that constitute the body. However, it is rather simple to obtain the dynamics for just the center of mass if we assume that the internal degrees of freedom are frozen and cannot be excited by the gravitational perturbation as in the case of a rigid body. In such an approximation it is convenient to define the center of mass ( $\hat{\mathbf{X}}$ ) and relative coordinate ( $\hat{\mathbf{r}}$ ) operators:

$$\begin{cases} \hat{\mathbf{X}} = \int d^3r \mathbf{r} \frac{\hat{m}(\mathbf{r})}{M} \\ \hat{\mathbf{r}} = \hat{\mathbf{x}} - \hat{\mathbf{X}} \end{cases} \quad (24)$$

and their canonical conjugates, respectively  $\hat{\mathbf{P}}$  and  $\hat{\mathbf{p}}_r$ , where  $\hat{m}(\mathbf{r})$  is the mass density operator [34] and  $M = \int d^3r \hat{m}(\mathbf{r})$  is the total mass. Upon tracing out the relative degrees of freedom, the Hamiltonian for the center of mass of a rigid body reads:

$$\begin{aligned} \hat{H} &= Mc^2 + \frac{\hat{\mathbf{P}}^2}{2M} + \int d^3r h^{00}(\mathbf{r}, t)m(\hat{\mathbf{X}} + \mathbf{r})c^2 + \\ &\quad - \int d^3r h^{00}(\mathbf{r}, t)\frac{m(\mathbf{r} + \hat{\mathbf{X}})}{M}\frac{\hat{\mathbf{P}}^2}{4M} + \\ &\quad + c \int d^3r h^{0i}(\mathbf{r}, t)\frac{m(\mathbf{r} + \hat{\mathbf{X}})}{M}\hat{P}_i + \\ &\quad - \int d^3r h^{ij}(\mathbf{r}, t)\frac{m(\mathbf{r} + \hat{\mathbf{X}})}{M}\frac{\hat{P}^i\hat{P}^j}{2M} + \\ &\quad + \frac{\hbar^2c^2}{8M} \int d^3r \nabla^2(\text{tr}[h^{\mu\nu}(\mathbf{r}, t)])\frac{m(\hat{\mathbf{X}} + \mathbf{r})}{M} + \\ &\quad + \frac{i\hbar c^2}{2} \int d^3r \partial_t \left( h^{00}(\mathbf{r}, t) - \frac{1}{2}\text{tr}(h^{\mu\nu}(\mathbf{r}, t)) \right) \frac{m(\hat{\mathbf{X}} + \mathbf{r})}{M} \end{aligned} \quad (25)$$

Eq. (25) was derived following the work of [29] where, however, the authors only consider the special case with

$$h^{0i} = h^{ij} = 0.$$

In the next section we will specialize to the case of a (weak) stochastic gravitational background.

#### IV. STOCHASTIC GRAVITATIONAL PERTURBATION: SINGLE PARTICLE MASTER EQUATION

The motivation to consider a stochastic weak gravitational perturbation is given by the works on stochastic gravity (see for example [30] for a review and further references) and by the interest in a stochastic gravitational background analogous to the cosmic microwave background radiation created in the early universe that has survived to the present era (see for instance [11, 12]). We also treat the gravitational perturbation as classical as a first approximation.

If the metric is random Eq. (20) becomes a stochastic differential equation. As a consequence the predictions are given by taking the stochastic average over the random gravitational field. We then need to specify its stochastic properties.

We assume the noise to be gaussian and with zero mean. The first assumption is justified by the law of large numbers, while the second by our choice of taking from the very beginning the Minkowski spacetime as the background spacetime around which the metric fluctuates. For the sake of simplicity, we also assume the different components of the metric fluctuation to be uncorrelated. This means that the noise is fully

characterized by:

$$\begin{aligned} \mathbb{E}[h_{\mu\nu}(\mathbf{x}, t)] &= 0 \\ \mathbb{E}[h_{\mu\nu}(\mathbf{x}, t)h_{\nu\rho}(\mathbf{y}, s)] &= \alpha^2 f_{\mu\rho}(\mathbf{x}, \mathbf{y}; t, s) \end{aligned} \quad (26)$$

where  $\mathbb{E}[\cdot]$  denotes the stochastic average and  $\alpha$  represents the strength of the gravitational fluctuations. The two point correlation function  $f(\mathbf{x}, \mathbf{y}; t, s)$  is further characterized by its correlation time  $\tau_c$  and the correlation length  $L$ .

We move to the density operator formalism [35]:

$$\hat{\Omega}(t) = |\phi(t)\rangle\langle\phi(t)| \quad (27)$$

As the only characterization of the noise is given by the stochastic average (Eq. (26)), we study the dynamics of the averaged operator:

$$\hat{\rho}(t) = \mathbb{E}[\hat{\Omega}(t)] \quad (28)$$

Let us consider the von Neumann equation for the averaged density matrix :

$$\begin{aligned} \partial_t \hat{\rho}(t) &= -\frac{i}{\hbar} [\hat{H}_0(t), \hat{\rho}(t)] - \frac{i}{\hbar} \mathbb{E}[\hat{H}_p(t), \hat{\Omega}(t)] \\ &\equiv \mathbb{E}[\mathfrak{L}[\hat{\Omega}(t)]] \end{aligned} \quad (29)$$

where  $\mathfrak{L}[\cdot]$  denotes the Liouville superoperator. Equation (29) is in general difficult to tackle, because of the stochastic average, but it can be solved perturbatively by means of the cumulant expansion [31]. With the further help of the gaussianity, zero mean, uncorrelation of different components, we can rewrite Eq. (29) in Fourier space [36] as:

$$\begin{aligned}
\partial_t \hat{\rho} = & -\frac{i}{\hbar} [\hat{H}_0, \hat{\rho}(t)] + \\
& -\frac{\alpha^2}{\hbar^8} \int \frac{d^3 q d^3 q'}{(2\pi)^3} \int_0^t dt_1 \tilde{f}^{00}(\mathbf{q}, \mathbf{q}'; t, t_1) \frac{m(\mathbf{q})m(\mathbf{q}')}{M^2} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \left( \frac{\hat{P}^2}{4M} + \frac{Mc^2}{2} \right), \left[ e^{i\mathbf{q}'\cdot\hat{\mathbf{X}}_{t_1}/\hbar} \left( \frac{\hat{P}^2}{4M} + \frac{Mc^2}{2} \right), \hat{\rho}(t) \right] \right] + \\
& -\frac{\alpha^2 c^2}{\hbar^8} \int \frac{d^3 q d^3 q'}{(2\pi)^3} \int_0^t dt_1 \tilde{f}^{0i}(\mathbf{q}, \mathbf{q}'; t, t_1) \frac{m(\mathbf{q})m(\mathbf{q}')}{M^2} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \hat{P}_i, \left[ e^{i\mathbf{q}'\cdot\hat{\mathbf{X}}_{t_1}/\hbar} \hat{P}_i, \hat{\rho}(t) \right] \right] + \\
& -\frac{\alpha^2}{\hbar^8} \int \frac{d^3 q d^3 q'}{(2\pi)^3} \int_0^t dt_1 \tilde{f}^{ij}(\mathbf{q}, \mathbf{q}'; t, t_1) \frac{m(\mathbf{q})m(\mathbf{q}')}{M^2} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \frac{\hat{P}_i \hat{P}_j}{2M}, \left[ e^{i\mathbf{q}'\cdot\hat{\mathbf{X}}_{t_1}/\hbar} \frac{\hat{P}_i \hat{P}_j}{2M}, \hat{\rho}(t) \right] \right] + \\
& -\frac{\alpha^2}{\hbar^8} \int \frac{d^3 q d^3 q'}{(2\pi)^3} \int_0^t dt_1 \tilde{f}_\mu^\mu(\mathbf{q}, \mathbf{q}'; t, t_1) \frac{\mathbf{q}^2 \mathbf{q}'^2}{64M^2} \frac{m(\mathbf{q})m(\mathbf{q}')}{M^2} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \left[ e^{i\mathbf{q}'\cdot\hat{\mathbf{X}}_{t_1}/\hbar}, \hat{\rho}(t) \right] \right] + \\
& -\frac{\alpha^2}{16\hbar^4} \int \frac{d^3 q d^3 q'}{(2\pi)^3} \int_0^t dt_1 \partial_t \partial_{t_1} \tilde{f}_\mu^\mu(\mathbf{q}, \mathbf{q}'; t, t_1) \frac{m(\mathbf{q})m(\mathbf{q}')}{M^2} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \left[ e^{i\mathbf{q}'\cdot\hat{\mathbf{X}}_{t_1}/\hbar}, \hat{\rho}(t) \right] \right] + O(t\alpha^3 \tau_c^2)
\end{aligned} \tag{30}$$

where  $\hat{x}_{t_1} = e^{i\hat{H}_0 t_1} \hat{x} e^{-i\hat{H}_0 t_1}$ . The above equation describes the dynamics of the rigid body's center of mass is in the presence of an external weak, stochastic gravitational field (with the further assumptions made in this section), and constitutes the main result of this paper.

In the following we will not consider the effect on the dynamics due to the derivatives of the metric perturbation, as in typical experimental situations [5–9] they are negligible and in any case they would not add any further informative content to the analysis. This means that we neglect the last line of Eq. (30).

We now restrict our analysis to the Markovian case, i.e. we assume the noise to be delta correlated in time ( $\tau_c \rightarrow 0$ ):

$$f^{\mu\nu}(\mathbf{x}, \mathbf{y}; t, s) = j^{\mu\nu}(\mathbf{x}, \mathbf{y}; t) \delta(t - s) \tag{31}$$

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A further reasonable assumption, motivated by the homogeneity of spacetime itself, is that of translational invariance of the two point correlation function:

$$f^{\mu\nu}(\mathbf{x}, \mathbf{y}; t, s) = \lambda u^{\mu\nu}(\mathbf{x} - \mathbf{y}) \delta(t - s) \tag{32}$$

where the factor  $\lambda$  is in principle a generic coefficient with the dimension of a time. We assume it to be

$$\lambda = \frac{L}{c} \tag{33}$$

as this is the only time scale of the system. Note that this choice does not affect the generality of the analysis as we leave  $u^{\mu\nu}(\mathbf{x} - \mathbf{y})$  unspecified. In such a regime Eq. (30) is exact and it is easy to show that it reduces to:

$$\begin{aligned}
\partial_t \hat{\rho} = & -\frac{i}{\hbar} [\hat{H}, \hat{\rho}(t)] + \\
& -\frac{\alpha^2 L}{4(2\pi)^3 \hbar^8 c} \int d^3 q \tilde{u}^{00}(\mathbf{q}) m^2(\mathbf{q}) \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \hat{\rho}(t) \right] \right] + \\
& -\frac{\alpha^2 L}{(2\pi)^3 \hbar^8 c} \int d^3 q \tilde{u}^{00}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M^2} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \frac{\hat{\mathbf{P}}^2}{2M}, \left[ e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \frac{\hat{\mathbf{P}}^2}{2M}, \hat{\rho}(t) \right] \right] + \\
& -\frac{\alpha^2 L}{2(2\pi)^3 \hbar^8 c} \int d^3 q \tilde{u}^{00}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \left[ e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \frac{\hat{\mathbf{P}}^2}{2M}, \hat{\rho}(t) \right] \right] + \\
& -\frac{\alpha^2 L}{2(2\pi)^3 \hbar^8 c} \int d^3 q \tilde{u}^{00}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \frac{\hat{\mathbf{P}}^2}{2M}, \left[ e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \hat{\rho}(t) \right] \right] + \\
& -\frac{\alpha^2 L c}{(2\pi)^3 \hbar^8} \int d^3 q \tilde{u}^{0i}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M^2} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \hat{P}_i, \left[ e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \hat{P}_i, \hat{\rho}(t) \right] \right] + \\
& -\frac{\alpha^2 L}{(2\pi)^3 \hbar^8 c} \int d^3 q \tilde{u}^{ij}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M^2} \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \frac{\hat{P}_i \hat{P}_j}{2M}, \left[ e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar} \frac{\hat{P}_i \hat{P}_j}{2M}, \hat{\rho}(t) \right] \right]
\end{aligned} \tag{34}$$

Eq. (34) describes decoherence both in position and in momentum, as it contains double commutators of functions of the position, momentum and free kinetic energy operators respectively with the averaged density matrix. In particular, we immediately recognize the term in the second line of Eq. (34) to give decoherence in position, that in the third line might give decoherence in energy (in the regime in which  $\frac{\mathbf{q}\cdot\hat{\mathbf{X}}}{\hbar}$  is small), and that in the sixth line decoherence in momentum (in the same regime).

In the next section we will investigate under which conditions Eq. (34) reduces the different models of gravitational decoherence present in the literature.

## V. DECOHERENCE IN THE POSITION EIGENBASIS

In this section we specialize Eq. (34) to the regime in which the dominant contribution to the decoherence effect is in the position eigenbasis. This can be done

under the following assumptions:

$$\begin{cases} h^{00} \gtrsim h^{0i} \\ h^{00} \gtrsim h^{ij} \\ \Delta E \ll M c^2 (1 - u^{00}(\Delta \mathbf{x})) \end{cases} \tag{35}$$

where  $\Delta \mathbf{x}$  and  $\Delta E$  are the quantum coherences of the system, respectively the position and energy ( $E = \frac{\mathbf{P}^2}{2M}$ ). It is then easy to show that the leading contribution to Eq. (34) is:

$$\begin{aligned}
\partial_t \hat{\rho} = & -\frac{i}{\hbar} [\hat{H}, \hat{\rho}(t)] + \\
& -\frac{\alpha^2 L c^3}{(2\pi)^3 \hbar^5} \int d^3 q \tilde{u}^{00}(\mathbf{q}) m^2(\mathbf{q}) \left[ e^{i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \left[ e^{-i\mathbf{q}\cdot\hat{\mathbf{X}}/\hbar}, \hat{\rho}(t) \right] \right] \\
& + O(\hbar^{\mu i}) + O(\Delta E)
\end{aligned} \tag{36}$$

The above equation describes decoherence in the position eigenbasis as the Lindblad operator is a function of the position operator. It is actually of the same form of the Gallis-Fleming master equation [32], which describes the decoherence induced on a particle by collisions with a surrounding thermal gas, allowing for a collisional interpretation of the result.

To compare with the previous literature on gravitational decoherence, we must further characterize the spatial correlation function of the noise and the mass density distribution. We start by considering the model proposed by



Blencowe [22]. In order to recover an analogous master equation we must assume the noise to be delta correlated in space:

$$u^{00}(\mathbf{x} - \mathbf{x}') = L^3 \delta^3(\mathbf{x} - \mathbf{x}') \quad (37)$$

Under this assumptions Eq. (34), represented in the position eigenbasis, in fact becomes:

$$\begin{aligned} \partial_t \rho(\mathbf{x}, \mathbf{x}'; t) = & \frac{i\hbar}{2M} (\nabla_x^2 - \nabla_{x'}^2) \rho(\mathbf{x}, \mathbf{x}'; t) + \\ & - \frac{(\alpha^{00})^2 c^3 L^4}{4\hbar^2} \int d^3 r \left( m(\mathbf{r} - \mathbf{x}) - m(\mathbf{r} - \mathbf{x}') \right)^2 \rho(\mathbf{x}, \mathbf{x}'; t) \\ & + O(\hbar^{\mu i}) \end{aligned} \quad (38)$$

which has the same form of the master equation obtained in [22], and describes decoherence in position. The different rate is due to the different treatment of the gravitational noise: Blencowe considers a quantum bosonic thermic bath whose correlation functions can not be reproduced by our classical description of the noise. If we further take the mass density function to be a gaussian:

$$m(\mathbf{r}) = \frac{m}{(\sqrt{2\pi}R)^3} e^{-\mathbf{r}^2/(2R^2)} \quad (39)$$

as it is done in the same work, Eq. (38) then reads:

$$\begin{aligned} \partial_t \rho(\mathbf{x}, \mathbf{x}'; t) = & \frac{i\hbar}{2M} (\nabla_x^2 - \nabla_{x'}^2) \rho(\mathbf{x}, \mathbf{x}'; t) + \\ & - \frac{\alpha^2 M^2 c^3 L^4}{4(\sqrt{\pi})^3 \hbar^2 R^3} \left( 1 - e^{-\frac{(\mathbf{x} - \mathbf{x}')^2}{4R^2}} \right) \rho(\mathbf{x}, \mathbf{x}'; t) \\ & + O(\hbar^{\mu i}) \end{aligned} \quad (40)$$

To recover the results obtained by Sanchez Gomez [20], we instead first need to take the mass density function to be ponitlike:

$$m(\mathbf{r}) = M \delta^3(\mathbf{r}) \quad (41)$$

as in [20], and then to assume the spatial correlation function to be gaussian:

$$\tilde{u}^{00}(\mathbf{q} - \mathbf{q}') = L^3 (\sqrt{2\pi}\hbar)^3 \delta(\mathbf{q} - \mathbf{q}') e^{-\hbar^2 \mathbf{q}^2 L^2 / 2} \quad (42)$$

With this choice for the spatial correlation functions Eq. (36) represented in the position basis reduces to:

$$\begin{aligned} \partial_t \rho(\mathbf{x}, \mathbf{x}'; t) = & \frac{i\hbar}{2m} (\nabla_x^2 - \nabla_{x'}^2) \rho(\mathbf{x}, \mathbf{x}'; t) + \\ & + \frac{2\alpha^2 m^2 c^3 L}{\hbar^2} \left( e^{-\frac{(\mathbf{x} - \mathbf{x}')^2}{2L^2}} - 1 \right) \rho(\mathbf{x}, \mathbf{x}'; t) \end{aligned} \quad (43)$$

and exactly recovers Sanchez Gomez's result.

A very similar equation was also obtained by Power and Percival [21]. To recover their result, we must consider again a point-like mass density and a gaussian spatial correlation function multiplied by a factor  $\sqrt{\pi}/2$ .

In the next section we will describe under which assumptions our model is able to describe decoherence in the momentum and energy eigenbasis thus encompassing the results of Breuer et al. [19] that predict gravitational decoherence to occur in the energy eigenbasis.

## VI. DECOHERENCE IN THE MOMENTUM EIGENBASIS

In this section we specialize Eq. (34) to the regime in which the dominant contribution to the decoherence effect is in the momentum or energy eigenbasis. This is the case when we can approximate:

$$e^{i\mathbf{q} \cdot \hat{\mathbf{X}}/\hbar} \sim \hat{1} \quad (44)$$

i.e. in the case of small  $\mathbf{q}$ . In this case Eq. (34) reduces to:

$$\begin{aligned} \partial_t \hat{\rho} = & - \frac{i}{\hbar} [\hat{H}, \hat{\rho}(t)] \\ & - \frac{\alpha^2 L}{(2\pi)^3 \hbar^8 c} \int d^3 q \tilde{u}^{00}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M^2} \left[ \frac{\hat{\mathbf{P}}^2}{2M}, \left[ \frac{\hat{\mathbf{P}}^2}{2M}, \hat{\rho}(t) \right] \right] \\ & - \frac{\alpha^2 L c}{(2\pi)^3 \hbar^8} \int d^3 q \tilde{u}^{0i}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M^2} \left[ \hat{P}_i, \left[ \hat{P}_i, \hat{\rho}(t) \right] \right] \\ & - \frac{\alpha^2 L}{(2\pi)^3 \hbar^8 c} \int d^3 q \tilde{u}^{ij}(\mathbf{q}) \frac{m^2(\mathbf{q})}{M^2} \left[ \frac{\hat{P}_i \hat{P}_j}{2M}, \left[ \frac{\hat{P}_i \hat{P}_j}{2M}, \hat{\rho}(t) \right] \right] \end{aligned} \quad (45)$$

In order to recover the results of Breuer et al. [19], the following hierarchy of the gravitational fluctuation must



be verified:

$$\begin{cases} h^{ij} \gg h^{0i} \\ h^{ij} \gg h^{00} \end{cases} \quad (46)$$

and the spatial correlation functions are chosen as follows:

$$\tilde{u}^{ij}(\mathbf{q} - \mathbf{q}') = \delta^{ij} L^3 (\sqrt{2\pi\hbar})^3 \delta(\mathbf{q} - \mathbf{q}') e^{-\hbar^2 \mathbf{q}^2 L^2 / 2} \quad (47)$$

and we take the mass density distribution to be Eq. (39) for the sake of simplicity and illustrative purposes. Under these assumptions Eq. (45) in fact reduces to:

$$\begin{aligned} \partial_t \hat{\rho} = & -\frac{i}{\hbar} [\hat{H}, \hat{\rho}(t)] + \\ & -\frac{\alpha^2 L^4}{\hbar^2 c (L^2 + 2R^2)^{3/2}} \left[ \frac{\hat{\mathbf{p}}^2}{2M}, \left[ \frac{\hat{\mathbf{p}}^2}{2M}, \hat{\rho}(t) \right] \right] \\ & + O(\hbar^{\mu 0}) \end{aligned} \quad (48)$$

Eq. (48) is indeed the same as the one obtained by Breuer et al. with the identification:

$$\frac{\alpha^2 L^4}{c(L^2 + 2R^2)^{3/2}} = T_c \quad (49)$$

where  $T_c$  is the spatially averaged correlation time of the noise present in the same paper [37].

## VII. CONCLUSIONS

In this paper we have derived a general model of decoherence for a non relativistic quantum particle interacting with a weak stochastic gravitational perturbation. We have specialized such an equation to the Markovian limit under some further reasonable assumptions on the stochastic properties of the gravitational noise motivated by simplicity arguments and cosmological models and observations.

We have extended our model to the description of the center of mass of a rigid extended body, which is a more realistic and experimentally interesting scenario.

Our Markovian master equation predicts decoherence in position, momentum and energy as it contains, among other terms, double commutators of functions of the position, momentum and free kinetic energy operators with

the averaged density matrix.

We were able to successfully recover other results present in the literature as appropriate limiting cases of our general master equation.

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## Appendix A: Feshbach Villars formalism

Here we provide explicit calculation for the derivation of Eq. (12).

Let us first rewrite Eq. (6) as:

$$\begin{aligned} (i\hbar\partial_t - i\hbar c h^{0i} \partial_i)^2 \psi = & \left[ \hbar^2 c \partial_t (h^{0i}) \partial_i - \hbar^2 c^2 (1 + h^{00}) \nabla^2 + \right. \\ & \left. - \hbar^2 c^2 h^{ij} \partial_i \partial_j + m^2 c^4 (1 + h^{00}) \right] \psi + \\ & + O(\hbar^2) \end{aligned} \quad (A1)$$

and the system of Eq. (8) as

$$\begin{cases} i\hbar(\partial_t - c h^{0i} \partial_i) \psi + m c^2 \psi = 2 m c^2 \phi \\ i\hbar(\partial_t - c h^{0i} \partial_i) \psi - m c^2 \psi = -2 m c^2 \chi \end{cases} \quad (A2)$$

Casting Eq. (A1) in the above system we get :

$$\begin{aligned} i\hbar(\partial_t - c h^{0i} \partial_i) \phi = & \frac{m c^2}{2} (\phi - \chi) + \\ & + \frac{m^2 c^4}{2 m c^2} (1 + h^{00}) (\phi + \chi) + \\ & - \frac{\hbar^2}{2 m} (1 + h^{00}) \nabla^2 (\phi + \chi) + \\ & - \frac{\hbar^2}{2 m} h^{ij} \partial_i \partial_j (\phi + \chi) + \\ & + \frac{\hbar^2}{2 m c} \partial_t (h^{0i}) \partial_i (\phi + \chi) \end{aligned} \quad (A3)$$

$$\begin{aligned}
i\hbar(\partial_t - ch^{0i}\partial_i)\chi = & -\frac{mc^2}{2}(\phi - \chi) + \\
& -\frac{m^2c^4}{2mc^2}(1 + h^{00})(\phi + \chi) + \\
& +\frac{\hbar^2}{2m}(1 + h^{00})\nabla^2(\phi + \chi) + \quad (\text{A4}) \\
& +\frac{\hbar^2}{2m}h^{ij}\partial_i\partial_j(\phi + \chi) + \\
& -\frac{\hbar^2}{2mc}\partial_t(h^{0i})\partial_i(\phi + \chi)
\end{aligned}$$

Recalling now that  $\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$  and exploiting the Pauli matrices, the system reduces to:

$$\begin{aligned}
i\hbar\partial_t\Psi = & \left[ mc^2\sigma_3 + \frac{mc^2}{2}h^{00}[\sigma_3 + i\sigma_2] + i\hbar ch^{0i}\partial_i \right. \\
& -\frac{\hbar^2}{2m}(1 + h^{00})[\sigma_3 + i\sigma_2]\nabla^2 + \\
& -\frac{\hbar^2}{2mc}\partial_t(h^{0i})[\sigma_3 + i\sigma_2]\partial_i + \quad (\text{A5}) \\
& \left. -\frac{\hbar^2}{2m}h^{ij}[\sigma_3 + i\sigma_2]\partial_i\partial_j \right]\Psi \\
= & : \mathfrak{H}\Psi
\end{aligned}$$

Upon applying the transformation (11), the EOM transform as:

$$\mathfrak{H} \rightarrow K := T\mathfrak{H}T^{-1} + i\hbar T\partial_t(T^{-1}) \quad (\text{A6})$$

and read exactly as Eq. (12) of the main text.

## Appendix B: Foldy Wouthuysen method

Here we illustrate the Foldy Wouthuysen method applied to Eq. (12). Let us consider the transformations:

$$K \rightarrow K' = U(K - i\hbar\partial_t)U^{-1} \quad (\text{B1})$$

and specialize  $U$  to Eq. (17), i.e.

$$U = e^{-i\sigma_3\mathcal{O}/(2mc^2)} =: e^{iS} \quad (\text{B2})$$

With the help of the BCH identity:

$$\begin{aligned}
K' = e^{iS}(K - i\hbar\partial_t)e^{-iS} = & K + i[S, K] + \frac{i^2}{2!}[S[S, K]] + \\
& +\frac{i^3}{3!}[S[S[S, K]]] + \dots \\
& +\hbar(-\dot{S} - \frac{i}{2}[S, \dot{S}] + \frac{1}{6}[S, [S, \dot{S}]] + \dots) \quad (\text{B3})
\end{aligned}$$

Recalling that:

$$K = mc^2\sigma_3 + \mathfrak{E} + \mathcal{O} \quad (\text{B4})$$

and noticing that:

$$[\sigma_3, \mathfrak{E}] = 0 \quad (\text{B5})$$

$$\{\sigma_3, \mathcal{O}\} = 0 \quad (\text{B6})$$

$$[\sigma_3\mathcal{O}, \sigma_3] = -2\mathcal{O} \quad (\text{B7})$$

$$[\sigma_3\mathcal{O}, \mathfrak{E}] = \sigma_3[\mathcal{O}, \mathfrak{E}] \quad (\text{B8})$$

$$[\sigma_3\mathcal{O}, \mathcal{O}] = 2\sigma_3\mathcal{O}^2 \quad (\text{B9})$$

it is not difficult to check that:

$$K' = mc^2\sigma_3 + \mathfrak{E}' + \mathcal{O}' \quad (\text{B10})$$

where:

$$\begin{aligned}
\mathfrak{E}' = & \mathfrak{E} + \sigma_3\left(\frac{\mathcal{O}^2}{2mc^2} - \frac{\mathcal{O}^4}{8m^3c^6}\right) - \frac{i}{8m^2c^4}[\mathcal{O}, \dot{\mathcal{O}}] \\
& - \frac{1}{8m^2c^4}[\mathcal{O}, [\mathcal{O}, \mathfrak{E}]] + \dots \quad (\text{B11})
\end{aligned}$$

$$\mathcal{O}' = \frac{1}{2mc^2}\sigma_3[\mathcal{O}, \mathfrak{E}] - \frac{\mathcal{O}^3}{3m^2c^4} + \frac{i}{2mc^2}\sigma_3\dot{\mathcal{O}} + \dots \quad (\text{B12})$$

We note that  $\mathcal{O}'$  is of order  $c^{-1}$ , meaning that we need to perform a further transformation if we want non trivial diagonal EOM. The transformation that we perform is:

$$U' = e^{-i\sigma_3\mathcal{O}'/(2mc^2)} \quad (\text{B13})$$

after which the Hamiltonian reads:

$$K'' = mc^2\sigma_3 + \mathfrak{E}' + \mathcal{O}'' + \dots \quad (\text{B14})$$

with:

$$\mathcal{O}'' = \frac{\sigma_3}{2mc^2}[\mathcal{O}', \mathfrak{E}'] + \frac{i}{2mc^2}\sigma_3\dot{\mathcal{O}}' + \dots \quad (\text{B15})$$

As  $\mathcal{O}'' \sim O(\frac{v^3}{c^3})$  we need to perform a final transformation:

$$U'' = e^{-i\sigma_3\mathcal{O}''/(2mc^2)} \quad (\text{B16})$$

Finally the Hamiltonian reads:

$$H := K''' = mc^2\sigma_3 + \mathfrak{E}' + O(c^{-4}) \quad (\text{B17})$$

It is easy to note that the only (other than  $\mathfrak{E}$ ) contribution to  $\mathfrak{E}'$  at the desired order is:

$$\begin{aligned} \frac{\sigma_3}{2mc^2} \mathcal{O}^2 &= \frac{\sigma_3}{2mc^2} \left\{ \frac{imc^2}{2} h^{00} \sigma_2, -\frac{i\hbar^2}{2m} \nabla^2 \sigma_2 \right\} + \\ &\quad + O(\hbar^2) + O(c^{-4}) \\ &= \frac{\hbar^2}{4m} (h^{00} \nabla^2 + \nabla^2 (h^{00})) \sigma_3 + O(\hbar_{\mu\nu}^2) + O(c^{-4}) \end{aligned} \quad (\text{B18})$$

so that the Hamiltonian becomes:

$$\begin{aligned} H &= mc^2 \left( 1 + \frac{h^{00}}{2} \right) \sigma_3 - \frac{\hbar^2}{2m} \left( 1 + \frac{h^{00}}{2} \right) \nabla^2 \sigma_3 + \\ &\quad - \frac{\hbar^2}{2m} h^{ij} \partial_i \partial_j \sigma_3 + i\hbar c h^{0i} \partial_i + \frac{i\hbar}{2} \partial_t (h^{00}) \\ &\quad - \frac{i\hbar}{4} \partial_t (\text{tr}(h^{\mu\nu})) + \frac{\hbar^2}{8m} \nabla^2 (\text{tr}(h^{\mu\nu})) \sigma_3 + \\ &\quad + O(c^{-4}) + O(\hbar_{\mu\nu}^2) \end{aligned} \quad (\text{B19})$$

as in Eq. (18) of the main text.

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- [33] The harmonic Gauge implies translational invariance of the infinitesimal volume in the chosen coordinate system as e.g. in cartesian coordinates.
- [34] Under the assumption that the rigid body consists in an ensemble of a large number  $N$  of particles, the density operator can be defined as  $\hat{m}(\mathbf{x}) = \sum_i \frac{m_i}{(2\pi\hbar)^3} \int d\mathbf{q} e^{-\frac{i}{\hbar}(\mathbf{x}-\hat{\mathbf{x}}_i)\cdot\mathbf{q}}$ , where  $m_i$  and  $\hat{x}_i$  are the mass and the position operator of the  $i$ -th particle.
- [35] We note that that of state vectors (and Schrödinger equation) is not the most appropriate formalism to adopt for the description of a quantum stochastic process in most experimental situation, as it does not allow one to describe statistical mixtures of quantum states.
- [36] Our choice for the Fourier transform is:
- $$f(\mathbf{x}) = \frac{1}{(\sqrt{2\pi\hbar})^3} \int d^3q \tilde{f}(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{x}/\hbar}$$
- [37] In the work of Breuer et al. the symbol used for the spatially averaged correlation time is  $\tau_c$ . It was here changed to  $T_c$  in order to avoid any confusion with our own notation.