

Testing CSL with neutron stars

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A recent paper [1] claims that the CSL model of spontaneous wave function collapse is ruled out by observations on heat flow from neutron stars. This type of system—a degenerate Fermi gas—is relevant as it represents the densest form of matter, potentially maximising CSL effects. As it turns out, this is not the case: to leading order, the CSL induced heating is the same as for ordinary matter, and neutron stars do not bound the CSL parameters significantly.

I. INTRODUCTION

Collapse models provide a phenomenological description of quantum measurements, by adding stochastic and non-linear terms to the Schrödinger equation, which implement the collapse of the wave function [2]. Such effects are negligible for microscopic systems, and become stronger when their mass increases. This is how the quantum-to-classical transition is described.

The most supported model is the Continuous Spontaneous Localization (CSL) model [3, 4], where the collapse effects are quantified by two parameters: the collapse rate λ , and the correlation length of the noise r_c . Different theoretical proposals for their numerical value were suggested: $\lambda = 10^{-16} \text{ s}^{-1}$ and $r_c = 10^{-7} \text{ m}$ by Ghirardi, Rimini and Weber [5]; $\lambda = 10^{-8 \pm 2} \text{ s}^{-1}$ for $r_c = 10^{-7} \text{ m}$, and $\lambda = 10^{-6 \pm 2} \text{ s}^{-1}$ for $r_c = 10^{-6} \text{ m}$ by Adler [6]. Experimental data were extensively used to bound the parameters [6–22] and new proposals were presented, suggesting how to further push these bounds [22–29]. Fig. 1 summarizes the state of the art.

A recent paper [1] takes a cosmological perspective to dig further in the CSL parameter space. A consequence of collapse models is a spontaneous heating, induced by the random collapse; based on this effect, the authors consider the balance between the CSL heating in neutron stars, whose extremely high density boosts the collapse effects, and the observed energy emitted as radiation. They then derive an upper bound that rules out all the plausible values of the collapse parameters.

Here, we re-analyze the problem, showing that this bound is weaker than other bounds already derived. Contrary to the results of [1], neutron stars are not the ideal place to test collapse models.

II. CSL MODEL - PERTURBATIVE CALCULATION

Following [21], we consider the transition amplitude $c_{fi}(t)$ caused by a perturbation, from an initial state $|i\rangle$ of a quantum system to a final state $|f\rangle$, with associated energies $E_i = \hbar\omega_i$ and $E_f = \hbar\omega_f$ respectively. For the sake of simplicity we restrict the problem to the case of one particle of mass m_A . The result for the N particle case is given in Appendix A. We have:

$$c_{fi}(t) = -\frac{i}{\hbar} \int_0^t ds \langle f | e^{\frac{i}{\hbar} \hat{H}_0 s} \hat{V}(s) e^{-\frac{i}{\hbar} \hat{H}_0 s} | i \rangle, \quad (1)$$

where \hat{H}_0 is the free Hamiltonian and the perturbation, for the CSL process applied to a particle of mass m_A , is [21]:

$$\begin{aligned} \hat{V}(t) &= \int d\mathbf{z} w_t(\mathbf{z}) \hat{\mathcal{V}}(\mathbf{z}), \\ \hat{\mathcal{V}}(\mathbf{z}) &= -\frac{\hbar}{m_0} m_A g(\mathbf{z} - \hat{\mathbf{x}}_A), \end{aligned} \quad (2)$$

where m_0 is the nucleon mass, $w_t(\mathbf{z})$ is a noise with zero mean ($\mathbb{E}[w_t(\mathbf{z})] = 0$) and correlator:

$$\mathbb{E}[w_t(\mathbf{z}) w_s(\mathbf{x})] = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \gamma(\omega) e^{-i\omega(t-s)} \delta(\mathbf{x} - \mathbf{z}), \quad (3)$$

where $\gamma(\omega) = \gamma(-\omega)$ is the frequency-dependent collapse strength. We denoted with $\hat{\mathbf{x}}_A$ the position operator of the particle, and:

$$g(\mathbf{x}) = \frac{e^{-\mathbf{x}^2/2r_c^2}}{(\sqrt{2\pi}r_c)^3} = \frac{1}{(2\pi)^3} \int d\mathbf{q} e^{-\mathbf{q}^2 r_c^2/2 - i\mathbf{q}\cdot\mathbf{x}}. \quad (4)$$

We assume that the particle is free and confined in a box of side L ; the initial and final states read:

$$\langle \mathbf{x} | i \rangle = \frac{e^{i\mathbf{k}_i \cdot \mathbf{x}}}{L^{3/2}}, \quad \text{and} \quad \langle \mathbf{x} | f \rangle = \frac{e^{i\mathbf{k}_f \cdot \mathbf{x}}}{L^{3/2}}. \quad (5)$$

We then have:

$$\begin{aligned} c_{fi}(t) &= \frac{im_A}{m_0 L^3} \int d\mathbf{q} e^{-\mathbf{q}^2 r_c^2/2} \int_0^t ds e^{i\omega_{fi}s} \\ &\quad \times \int d\mathbf{z} w_s(\mathbf{z}) e^{-i\mathbf{q}\cdot\mathbf{z}} \delta(\mathbf{k}_f - \mathbf{k}_i - \mathbf{q}), \end{aligned} \quad (6)$$

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where $\omega_{fi} = \omega_f - \omega_i$ and $\mathbf{k}_i, \mathbf{k}_f$ are the initial and final momenta of the particle, respectively. The transition

$$\mathbb{E}[|c_{fi}|^2] = \frac{m_A^2}{m_0^2 L^3} \int d\mathbf{q} e^{-\mathbf{q}^2 r_c^2} \int d\omega \gamma(\omega) \delta(\mathbf{k}_f - \mathbf{k}_i - \mathbf{q}) t \delta^{(t)}(\omega_{fi} - \omega), \quad (7)$$

where we used the relations:

$$\begin{aligned} [\delta(\mathbf{k}_f - \mathbf{k}_i - \mathbf{q})]^2 &\sim (L/(2\pi))^3 \delta(\mathbf{k}_f - \mathbf{k}_i - \mathbf{q}), \\ \int_0^t ds e^{i(\omega_{fi} - \omega)s} &= 2\pi e^{i(\omega_{fi} - \omega)t/2} \delta^{(t)}(\omega_{fi} - \omega), \quad (8) \\ [\delta^{(t)}(\omega_{fi} - \omega)]^2 &\sim (t/(2\pi)) \delta^{(t)}(\omega_{fi} - \omega). \end{aligned}$$

We now apply Eq. (7) to the system under study, i.e. a particle in a Fermi gas. The heating power $P_{\text{CSL}}(t) = dE_{\text{TOT}}(t)/dt$ reads:

$$P_{\text{CSL}}(t) = \frac{d}{dt} \sum_i \sum_f \mathcal{N}(\mathbf{k}_i) (1 - \mathcal{N}(\mathbf{k}_f)) \hbar \omega_{fi} \mathbb{E}[|c_{fi}(t)|^2], \quad (9)$$

where $\mathcal{N}(\mathbf{k}_i)$ is the probability of the initial state having momentum \mathbf{k}_i , and $(1 - \mathcal{N}(\mathbf{k}_f))$ is the probability for the final state with momentum \mathbf{k}_f not to be occupied, otherwise the particle could not jump there because of the Pauli exclusion principle. Since $\mathcal{N}(\mathbf{k}_i)\mathcal{N}(\mathbf{k}_f)$ and $\mathbb{E}[|c_{fi}|^2]$ are even, whereas ω_{fi} is odd, under the interchange $i \leftrightarrow f$, the term containing $\mathcal{N}(\mathbf{k}_i)\mathcal{N}(\mathbf{k}_f)$ makes a vanishing contribution to Eq. (9). The above expression then simplifies to

$$P_{\text{CSL}}(t) = \frac{d}{dt} \sum_i \sum_f \mathcal{N}(\mathbf{k}_i) \hbar \omega_{fi} \mathbb{E}[|c_{fi}(t)|^2]. \quad (10)$$

Using the standard box-normalization prescription, according to which in the limit $L \rightarrow +\infty$:

$$\frac{1}{L^3} \sum_{\mathbf{p}} g(\mathbf{p}) \rightarrow \frac{1}{(2\pi)^3} \int d\mathbf{p} g(\mathbf{p}), \quad (11)$$

one obtains

$$P_{\text{CSL}}(t) = \frac{L^3}{(2\pi)^3} \frac{d}{dt} \sum_i \mathcal{N}(\mathbf{k}_i) \int d\mathbf{k}_f \hbar \omega_{fi} \mathbb{E}[|c_{fi}(t)|^2], \quad (12)$$

which in the long time limit reads

$$P_{\text{CSL}}(t) = \frac{m_A^2}{m_0^2 (2\pi)^3} \sum_i \mathcal{N}(\mathbf{k}_i) \int d\mathbf{q} \hbar \bar{\omega}_i(\mathbf{q}) e^{-\mathbf{q}^2 r_c^2} \gamma(\bar{\omega}_i(\mathbf{q})), \quad (13)$$

where

$$\bar{\omega}_i(\mathbf{q}) = \frac{\hbar}{2m_A} (\mathbf{q}^2 + 2\mathbf{k}_i \cdot \mathbf{q}). \quad (14)$$

probability, under stochastic average, is then given by

In the white noise case, where $\gamma(\omega) = \gamma$, the integration over \mathbf{q} can be easily performed, giving:

$$P_{\text{CSL}}(t) = \frac{3}{4} \frac{\hbar^2 \lambda m_A}{m_0^2 r_c^2}, \quad (15)$$

where we used $\gamma = \lambda(\sqrt{4\pi}r_c)^3$ and $\sum_i \mathcal{N}(\mathbf{k}_i) = 1$. For the N atom case, the calculation of Appendix A shows that m_A in Eq. (15) is replaced by the total mass $M = Nm_A$. This is the same result obtained from the study of phononic modes in matter [21, 30, 31].

III. NEUTRON STARS

Neutron stars are small (radius ~ 10 km) and dense (mass $M \sim 1.4 - 4.2 \times 10^{30}$ kg and density $\mu \sim 10^{17}$ kg/m³), resulting from the collapsed cores of stars with mass above the Chandrasekhar limit [32]. After a first stage next to their formation, where they cool through emission of baryonic matter, the main cooling process is dominated by thermal emission of radiation [33, 34], which is described by the Stefan-Boltzmann law:

$$P_{\text{rad}} = S\sigma T^4, \quad (16)$$

where S is the surface of the neutron star, $\sigma = 5.6 \times 10^{-8}$ W m⁻²K⁻⁴ is the Stefan's constant and T is the effective black-body temperature of the star. As a reference value for the temperature we can consider $T = 0.28_{-0.12}^{+0.19} \times 10^6$ K, which refers to the neutron star PSR J 1840-1419 [35]. The radius is $R = 10$ km and the mass $M = 2 \times 10^{30}$ kg, equal to the solar mass, giving a density $\mu = 4.8 \times 10^{17}$ kg/m³. Variation of R and M , for typical dimensions of a neutron star, do not imply significant changes in the bounds on the CSL parameters.

IV. RESULTS AND DISCUSSION

Assuming that the neutron star's thermal radiation emission is balanced by the heating effect due to the CSL noise, we impose $P_{\text{rad}} = P_{\text{CSL}}$. This gives an estimate of collapse rate:

$$\lambda = \frac{16R^2 m_0^2 \pi r_c^2 T^4 \sigma}{3M\hbar^2}, \quad (17)$$

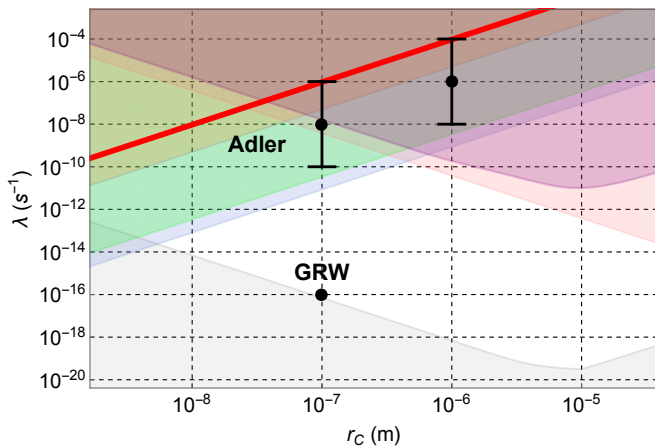


FIG. 1: Bounds on the collapse parameters λ and r_c for the standard (white noise) mass-proportional CSL model. The red line denotes the upper bound given by Eq. (17) applied to the heat flow from a neutron star. The shaded areas show the already experimentally and theoretically excluded regions: the orange region comes from cold atom experiment [13, 36]; the green region from phonon analysis in cryogenic experiments [21, 31, 37]; the blue region from x-ray emission from germanium [9–11, 19]; the purple region from mechanical cantilever [14, 17]; the pink region from LISA Pathfinder [15, 22, 38, 39]; the grey region from theoretical arguments [18, 20].

where we assumed that the neutron star can be approximated by a sphere of radius R . The corresponding upper bound is shown in red in Fig. 1; this turns out to be weaker than already existing bounds, and is further weakened if one assumes a low-frequency cut off in the noise spectrum following the methods of [21, 40].

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Appendix A: Field-theoretical calculation

We perform the same analysis presented in the main text, within the framework of quantum field theory. Let us consider the CSL Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{V}_{\text{CSL}}, \quad (\text{A1})$$

where

$$\hat{H}_0 = \sum_i \sum_\tau \sum_{\mathbf{p}} E_{\mathbf{p}\tau i} \hat{b}_{\mathbf{p}\tau i}^\dagger(t) \hat{b}_{\mathbf{p}\tau i}(t), \quad (\text{A2})$$

is the free Hamiltonian; the first sum is over the i -type of particle, the second sum over the spin (i -th type of particle) and the third over momentum. The CSL stochastic potential is [9]:

$$\hat{V}_{\text{CSL}} = -\hbar\sqrt{\gamma} \sum_j \sum_\sigma \frac{m_j}{m_0} \int d\mathbf{x} \hat{\Psi}_{\sigma j}^\dagger(\mathbf{x}, t) \hat{\Psi}_{\sigma j}(\mathbf{x}, t) \xi(\mathbf{x}, t), \quad (\text{A3})$$

Here we introduced:

$$\xi(\mathbf{x}, t) = \int d\mathbf{y} \frac{e^{-(\mathbf{x}-\mathbf{y})^2/2r_c^2}}{(\sqrt{2\pi}r_c)^3} w_t(\mathbf{y}), \quad (\text{A4})$$

whose mean and correlator are:

$$\mathbb{E}[\xi(\mathbf{x}, t)] = 0, \quad \text{and} \quad \mathbb{E}[\xi(\mathbf{x}, t)\xi(\mathbf{y}, s)] = \delta(t-s)F(\mathbf{x}-\mathbf{y}), \quad (\text{A5})$$

where \mathbb{E} denotes the stochastic average over the noise and $F(\mathbf{x}) = e^{-\mathbf{x}^2/4r_c^2}/(\sqrt{4\pi}r_c)^3$. The relation between the operator $\hat{\Psi}_{\sigma j}(\mathbf{x}, t)$ and $\hat{b}_{\mathbf{p}\tau i}(t)$ is given by

$$\begin{aligned} \hat{\Psi}_{\sigma j}(\mathbf{x}, t) &= \sum_{\mathbf{p}} \psi_{\mathbf{p}\sigma j}(\mathbf{x}) \hat{b}_{\mathbf{p}\sigma j}(t), \\ \hat{b}_{\mathbf{p}\sigma j}(t) &= \int d\mathbf{x} \psi_{\mathbf{p}\sigma j}^*(\mathbf{x}) \hat{\Psi}_{\sigma j}(\mathbf{x}, t), \end{aligned} \quad (\text{A6})$$

with $\psi_{\mathbf{p}\sigma j}(\mathbf{x})$ denoting the wavefunction for the single particle of type j , spin τ and of momentum \mathbf{p} . Below we specify the exact form of $\psi_{\mathbf{p}\sigma j}(\mathbf{x})$. The evolution of $\hat{b}_{\mathbf{p}\tau i}(t)$ is determined by the Heisenberg equation $\frac{d\hat{b}_{\mathbf{p}\tau j}(t)}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{b}_{\mathbf{p}\tau j}(t)]$, which gives

$$\frac{d\hat{b}_{\mathbf{p}\tau j}(t)}{dt} = -\frac{i}{\hbar}E_{\mathbf{p}\tau j}\hat{b}_{\mathbf{p}\tau j}(t) + i\sqrt{\gamma}\frac{m_j}{m_0} \sum_{\mathbf{k}} \int d\mathbf{x} \psi_{\mathbf{p}\tau j}^*(\mathbf{x}) \psi_{\mathbf{k}\tau j}(\mathbf{x}) \xi(\mathbf{x}, t) \hat{b}_{\mathbf{k}\tau j}(t). \quad (\text{A7})$$

The solution is:

$$\hat{b}_{\mathbf{p}\tau j}(t) = e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}t} \hat{b}_{\mathbf{p}\tau j}(0) + i\sqrt{\gamma}\frac{m_j}{m_0} \sum_{\mathbf{k}} \int_0^t d\mathbf{x} \psi_{\mathbf{p}\tau j}^*(\mathbf{x}) \psi_{\mathbf{k}\tau j}(\mathbf{x}) \int_0^t ds e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}(t-s)} \xi(\mathbf{x}, s) \hat{b}_{\mathbf{k}\tau j}(s). \quad (\text{A8})$$

Since $\hat{b}_{\mathbf{k}\tau j}(s)$ appears also in the last term, we need to solve perturbatively. We replace $\hat{b}_{\mathbf{k}\tau j}(s)$ with the corresponding form given again by Eq. (A8), and truncate the expression to order γ :

$$\hat{b}_{\mathbf{p}\tau j}(t) = \hat{A}_{\mathbf{p}\tau j}(t) + \hat{B}_{\mathbf{p}\tau j}(t) + \hat{C}_{\mathbf{p}\tau j}(t) + \mathcal{O}(\gamma^{3/2}), \quad (\text{A9})$$

where

$$\begin{aligned}
\hat{A}_{\mathbf{p}\tau j}(t) &= e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}t}\hat{b}_{\mathbf{p}\tau j}(0), \\
\hat{B}_{\mathbf{p}\tau j}(t) &= i\sqrt{\gamma}\frac{m_j}{m_0}\sum_{\mathbf{k}}\int d\mathbf{x}\psi_{\mathbf{p}\tau j}^*(\mathbf{x})\psi_{\mathbf{k}\tau j}(\mathbf{x})\int_0^t ds e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}(t-s)}\xi(\mathbf{x},s)e^{-\frac{i}{\hbar}E_{\mathbf{k}\tau j}s}\hat{b}_{\mathbf{k}\tau j}(0), \\
\hat{C}_{\mathbf{p}\tau j}(t) &= -\gamma\left(\frac{m_j}{m_0}\right)^2\sum_{\mathbf{k}\mathbf{k}'}\int d\mathbf{x}\psi_{\mathbf{p}\tau j}^*(\mathbf{x})\psi_{\mathbf{k}\tau j}(\mathbf{x})\int_0^t ds e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}(t-s)}\xi(\mathbf{x},s)\int d\mathbf{y}\psi_{\mathbf{k}\tau j}^*(\mathbf{y})\psi_{\mathbf{k}'\tau j}(\mathbf{y})\times \\
&\quad \int_0^s ds' e^{-\frac{i}{\hbar}E_{\mathbf{k}\tau j}(s-s')}\xi(\mathbf{y},s')e^{-\frac{i}{\hbar}E_{\mathbf{k}'\tau j}s'}\hat{b}_{\mathbf{k}'\tau j}(0).
\end{aligned} \tag{A10}$$

Due to the properties of the stochastic noise, Eq. (A5), the last term can be simplified to:

$$\hat{C}_{\mathbf{p}\tau j}(t) = -\frac{\gamma}{2(\sqrt{4\pi}r_c)^3}\left(\frac{m_j}{m_0}\right)^2\sum_{\mathbf{k}}\int d\mathbf{x}\psi_{\mathbf{p}\tau j}^*(\mathbf{x})\psi_{\mathbf{k}\tau j}(\mathbf{x})\int_0^t ds e^{-\frac{i}{\hbar}E_{\mathbf{p}\tau j}(t-s)}e^{-\frac{i}{\hbar}E_{\mathbf{k}\tau j}s}\hat{b}_{\mathbf{k}\tau j}(0). \tag{A11}$$

Given these expressions, we can compute the evolution of the energy expectation value, which is given by

$$E_{\text{TOT}}(t) = \mathbb{E}[\langle \hat{H} \rangle]. \tag{A12}$$

Due to the stochastic properties in Eq. (A5), we have $\mathbb{E}[\hat{V}_{\text{CSL}}] = 0$, therefore only \hat{H}_0 contributes to $E_{\text{TOT}}(t)$. In particular

$$E_{\text{TOT}}(t) = E_{\text{TOT}}(0) + E_{\text{TOT}}^{\text{CSL},1}(t) + E_{\text{TOT}}^{\text{CSL},2}(t) + \mathcal{O}(\gamma^{3/2}), \tag{A13}$$

where

$$\begin{aligned}
E_{\text{TOT}}(0) &= \sum_i \sum_{\tau} \sum_{\mathbf{p}} E_{\mathbf{p}\tau i} \langle \hat{A}_{\mathbf{p}\tau i}^\dagger(t) \hat{A}_{\mathbf{p}\tau i}(t) \rangle, \\
E_{\text{TOT}}^{\text{CSL},1}(t) &= \sum_i \sum_{\tau} \sum_{\mathbf{p}} E_{\mathbf{p}\tau i} \langle \hat{B}_{\mathbf{p}\tau i}^\dagger(t) \hat{B}_{\mathbf{p}\tau i}(t) \rangle, \\
E_{\text{TOT}}^{\text{CSL},2}(t) &= \sum_i \sum_{\tau} \sum_{\mathbf{p}} E_{\mathbf{p}\tau i} \langle \hat{A}_{\mathbf{p}\tau i}^\dagger(t) \hat{C}_{\mathbf{p}\tau i}(t) + \text{H.C.} \rangle,
\end{aligned} \tag{A14}$$

where H.C. stands for hermitian conjugate. We notice that there is no contribution from terms like $\hat{A}_{\mathbf{p}\tau i}^\dagger(t) \hat{B}_{\mathbf{p}\tau i}(t)$ or $\hat{B}_{\mathbf{p}\tau i}^\dagger(t) \hat{C}_{\mathbf{p}\tau i}(t)$: the first is zero under stochastic average and the second scales with $\gamma^{3/2}$ and can be then neglected. The above expressions, together with Eq. (A10), give:

$$\begin{aligned}
E_{\text{TOT}}(0) &= \sum_i \sum_{\tau} \sum_{\mathbf{p}} E_{\mathbf{p}\tau i} \langle \hat{b}_{\mathbf{p}\tau i}^\dagger(0) \hat{b}_{\mathbf{p}\tau i}(0) \rangle, \\
E_{\text{TOT}}^{\text{CSL},1}(t) &= \gamma \sum_i \sum_{\tau} \sum_{\mathbf{p}} E_{\mathbf{p}\tau i} \left(\frac{m_i}{m_0}\right)^2 \sum_{\mathbf{k}\mathbf{k}'} \int d\mathbf{x} \int d\mathbf{y} \psi_{\mathbf{p}\tau i}(\mathbf{x}) \psi_{\mathbf{k}\tau i}^*(\mathbf{x}) \psi_{\mathbf{p}\tau i}^*(\mathbf{y}) \psi_{\mathbf{k}\tau i}(\mathbf{y}) F(\mathbf{x} - \mathbf{y}) \times \\
&\quad \int_0^t ds e^{-\frac{i}{\hbar}(E_{\mathbf{k}\tau i} - E_{\mathbf{k}'\tau i})s} \langle \hat{b}_{\mathbf{k}\tau i}^\dagger(0) \hat{b}_{\mathbf{k}'\tau i}(0) \rangle, \\
E_{\text{TOT}}^{\text{CSL},2}(t) &= -\frac{\gamma}{(\sqrt{4\pi}r_c)^3} \sum_i \sum_{\tau} \sum_{\mathbf{p},\mathbf{k}} E_{\mathbf{p}\tau i} \left(\frac{m_i}{m_0}\right)^2 \int d\mathbf{x} \psi_{\mathbf{p}\tau i}^*(\mathbf{x}) \psi_{\mathbf{k}\tau i}(\mathbf{x}) \int_0^t ds e^{\frac{i}{\hbar}(E_{\mathbf{p}\tau i} - E_{\mathbf{k}\tau i})s} \langle \hat{b}_{\mathbf{p}\tau i}^\dagger(0) \hat{b}_{\mathbf{k}\tau i}(0) \rangle.
\end{aligned} \tag{A15}$$

The above terms contain $\langle \hat{b}_{\mathbf{p}\tau i}^\dagger(0) \hat{b}_{\mathbf{k}\tau i}(0) \rangle$. To compute it we consider a state of N particles with density matrix diagonal in momentum and weight given by the occupation number $\mathcal{N}(\mathbf{k})$. Then we have

$$\langle \hat{b}_{\mathbf{p}\tau i}^\dagger(0) \hat{b}_{\mathbf{k}\tau i}(0) \rangle = \delta_{\mathbf{p}\mathbf{k}} \mathcal{N}(\mathbf{p}). \tag{A16}$$

Applying this result we obtain

$$\begin{aligned}
E_{\text{TOT}}(0) &= \sum_i \sum_\tau \sum_{\mathbf{p}} E_{\mathbf{p}\tau i} \mathcal{N}(\mathbf{p}), \\
E_{\text{TOT}}^{\text{CSL},1}(t) &= \lambda t (2\sqrt{\pi} r_c)^3 \sum_i \sum_\tau \sum_{\mathbf{p}\mathbf{k}} E_{\mathbf{p}\tau i} \left(\frac{m_i}{m_0}\right)^2 \mathcal{N}(\mathbf{k}) \int d\mathbf{x} \int d\mathbf{y} \psi_{\mathbf{p}\tau i}(\mathbf{x}) \psi_{\mathbf{k}\tau i}^*(\mathbf{x}) \psi_{\mathbf{p}\tau i}^*(\mathbf{y}) \psi_{\mathbf{k}\tau i}(\mathbf{y}) F(\mathbf{x} - \mathbf{y}), \quad (\text{A17}) \\
E_{\text{TOT}}^{\text{CSL},2}(t) &= -\lambda t \sum_i \sum_\tau \sum_{\mathbf{p}} E_{\mathbf{p}\tau i} \left(\frac{m_i}{m_0}\right)^2 \mathcal{N}(\mathbf{p}),
\end{aligned}$$

where we substituted $\gamma = \lambda(2\sqrt{\pi} r_c)^3$.

So far the result is general. We now apply it to the case of interest, i.e. N particles in a cube box of length L . We apply the periodic boundary conditions and the box-normalization prescription

$$\psi_{\mathbf{p}\tau i}(\mathbf{x}) \rightarrow \phi_{\mathbf{q}\tau i}(\mathbf{x}) = \frac{e^{i\mathbf{q}\tau i \cdot \mathbf{x}}}{L^{3/2}}, \quad \text{with} \quad \mathbf{q}\tau i = \frac{2\pi}{L} \mathbf{n}\tau i, \quad (\text{A18})$$

where $\mathbf{n}\tau i \in \mathbb{Z}^3$. The wavefunctions $\phi_{\mathbf{q}\tau i}(\mathbf{x})$ are orthonormal

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} d\mathbf{x} \phi_{\mathbf{q}}(\mathbf{x}) \phi_{\mathbf{q}'}^*(\mathbf{x}) = \delta_{\mathbf{q}\mathbf{q}'}. \quad (\text{A19})$$

In the $L \rightarrow +\infty$ limit (so that space-integrals extend over the whole space and can be performed exactly) we have

$$E_{\text{TOT}}^{\text{CSL},1}(t) = \frac{8\lambda t \pi^{3/2} r_c^3}{L^3} \sum_i \sum_\tau \sum_{\mathbf{p}\mathbf{k}} E_{\mathbf{p}\tau i} \mathcal{N}(\mathbf{k}) e^{-(\mathbf{p}-\mathbf{k})^2 r_c^2} \left(\frac{m_i}{m_0}\right)^2. \quad (\text{A20})$$

The CSL heating power $P_{\text{CSL}} = \frac{d}{dt} E_{\text{TOT}}(t)$ is then given by

$$P_{\text{CSL}} = \lambda \sum_i \sum_\tau \sum_{\mathbf{p}} \left(\frac{m_i}{m_0}\right)^2 \mathcal{N}(\mathbf{p}) \left(\frac{8\pi^{3/2} r_c^3}{L^3} \sum_{\mathbf{k}} E_{\mathbf{k}\tau i} e^{-(\mathbf{p}-\mathbf{k})^2 r_c^2} - E_{\mathbf{p}\tau i} \right). \quad (\text{A21})$$

By taking $E_{\mathbf{k}\tau i} = \hbar^2 \mathbf{k}^2 / (2m_i)$ we find

$$\frac{8\pi^{3/2} r_c^3}{L^3} \sum_{\mathbf{k}} E_{\mathbf{k}\tau i} e^{-(\mathbf{p}-\mathbf{k})^2 r_c^2} \xrightarrow{L \rightarrow +\infty} \frac{3\hbar^2}{4m_i r_c^2} + \frac{\hbar^2 \mathbf{p}^2}{2m_i}. \quad (\text{A22})$$

By merging with Eq. (A21) we have:

$$P_{\text{CSL}} = \frac{3\hbar^2 \lambda}{4m_0^2 r_c^2} \sum_i m_i \sum_\tau \sum_{\mathbf{p}} \mathcal{N}(\mathbf{p}) = \frac{3\hbar^2 \lambda M}{4m_0^2 r_c^2}, \quad (\text{A23})$$

since that $\sum_\tau \sum_{\mathbf{p}} \mathcal{N}(\mathbf{p})$ gives the number of particle of type i .